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The Determination of Real Interest Rates

by

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Abstract

This paper reviews recent developments in the theory of interest-rate determination in closed and open economies. It addresses five basic questions: (1) what determines real interest rates in the steady state; (2) what can explain the movements in real interest rates over the last 20 years; (3) can government policies in a small open economy influence rates; (4) do after-tax rates below the real economic growth rate indicate that government debt should be increased, and (5) what are the implications of recent interest rate behaviour for government policy.

It is found that the equilibrium real interest rate depends on both tastes and technology. In terms of tastes, the important factors are: the rate of time preference, the length of the planning horizon of households, and the elasticity of intertemporal substitution. The technological factors affecting real interest rates are: the growth rate of labour productivity, the share of output going to various demographic groups, and the variability of income flows. Governments can affect real interest rates through the level of the government debt, tax and regulatory policies, and the choice of the long-run rate of inflation.

There is no "unique" real interest rate: risk, liquidity, and tax considerations create gaps between real interest rates on different types of assets. Real interest rates on government debt were historically low in the 1970s and high in the 1980s, while the cost of capital was more stable over this period. Three factors may have contributed to this experience. The oil-price shocks of the 1970s and the subsequent policy responses may have increased uncertainty about future income flows, lowering the risk-free rate of interest and raising the risk premium on private-sector debt. Rising inflation in the 1970s may also have reduced the yield on (relatively liquid) government bonds in that decade; the accompanying financial-market liberalisation probably reduced the cost of funds to firms and households in the 1980s, raising the yield on government debt while lowering it for the private sector. The very high real interest rates at the beginning of the 1980s are attributable to the temporary effects of tight monetary policy.

The gap between the real interest rate and the real economic growth rate can have important implications for government policy. If the gap is negative and the marginal product of capital equals the real interest rate, the economy is "dynamically-inefficient": both current and future welfare can be increased by raising the government debt and crowding out a portion of the existing capital stock. The evidence for dynamic inefficiency, however, is unfavourable. Even in periods when the real interest rate on government debt has been less than the real economic growth rate, tax wedges and risk premiums mean that the marginal product of capital has likely exceeded the real growth rate. The true cost of government debt, then, is not the rate of interest on the debt, but the marginal product of the capital stock that is crowded out by higher government debt.

In a small open economy, government debt tends to affect the net foreign asset position rather than the domestic capital stock. Nevertheless, the government can influence domestic interest rates to the extent that assets are not perfect substitutes internationally; a rise in government debt will increase net foreign debt, raising the yield required by investors to hold the debt. Tax considerations can also cause divergences between domestic and foreign interest rates, as can the interaction between differences in inflation rates between countries and non-indexed tax systems. Finally, if the demand curve for home-country exports is not perfectly elastic, a higher external debt requires a terms-of-trade deterioration in the steady state; this raises the true cost of government borrowing above the world rate of interest regardless of the degree on international asset substitutability.

Looking to the future, it appears that two important factors have contributed to the current high real interest rates on government debt, while

at the same time increasing the access to capital markets of private-sector agents and lowering their cost of borrowing: these factors are financial-market liberalisation and low rates of inflation. It seems unlikely, then, that the spread between public-sector and private-sector costs of borrowing will return to historical levels in the future, unless inflation rises or financial-market liberalisation is reversed. Regarding the overall level of interest rates, given no other changes, there could be future upward pressure from demographic changes and/or a failure to arrest the rising trend in government debt.

RÉSUMÉ

Ce document traite des progrès récents de la théorie sur l'établissement des taux d'intérêt selon qu'il s'agisse d'économies ouvertes ou fermées. Les cinq grandes questions suivantes sont abordées : (1) quels facteurs déterminent les taux d'intérêt dans un état stable; (2) comment expliquer l'évolution des taux d'intérêt au cours des 20 dernières années; (3) les politiques gouvernementales d'un petit pays à économie ouverte peuvent-elles influer sur les taux; (4) le fait que les taux après impôt soient inférieurs à la croissance économique réelle signifie-t-il que la dette publique devrait être accrue; et (5) quelles sont les répercussions de l'évolution récente des taux d'intérêt sur les politiques gouvernementales.

On constate que l'équilibre des taux d'intérêt réels dépend des goûts et de la technologie. Dans le premier cas, les facteurs importants sont les suivants : la fréquence des études, la période de planification adoptée par les ménages et l'élasticité de la substitution intertemporelle. Quant aux aspects technologiques influant sur les taux d'intérêt, citons le taux de croissance de la productivité de la main-d'oeuvre, la répartition de la production entre les groupes démographiques et le caractère variable des mouvements des revenus. De leur côté, les gouvernements peuvent avoir une incidence sur les taux d'intérêt réels par le biais du niveau de la dette publique, des politiques fiscales et de la réglementation et de leur choix concernant le taux d'inflation à long terme.

Il n'existe pas de taux d'intérêt réel «unique». Les risques, le niveau de liquidité et les considérations fiscales engendrent des écarts entre les taux d'intérêt réels visant différentes catégories d'éléments d'actif. Les taux d'intérêt réels sur la dette publique étaient modestes dans les années 70 et élevés au cours des années 80, tandis que le coût du capital était plus stable à cette époque. Trois facteurs peuvent avoir contribué à cette situation. Les bouleversements du prix du pétrole dans les années 70 et les réactions qu'ils ont suscité ont peut-être alimenté les inquiétudes au sujet des mouvements futurs des recettes, d'où une baisse du taux d'intérêt sans risques et une hausse de la prime de risque visant la dette du secteur privé. L'accroissement de l'inflation dans les années 70 peut avoir limité le rendement des obligations des gouvernements, qui sont relativement faciles à liquider, au cours de cette décennie. En outre, la libéralisation des marchés financiers a probablement réduit le coût des emprunts des ménages et des entreprises dans les années 80, contribuant à l'amélioration du rendement de la dette publique et à sa détérioration dans le cas du secteur privé. Les taux d'intérêt réels très élevés constatés au début des années 80 résultent des effets temporaires d'une politique monétaire ferme.

L'écart entre le taux d'intérêt et le taux de croissance réels de l'économie peut avoir d'importantes répercussions sur la politique du gouvernement. Si l'écart est négatif et si le rendement marginal du capital correspond au taux d'intérêt réel, l'économie est «dynamiquement inefficiente» : la prospérité actuelle et future peut être accrue en augmentant la dette publique et en évinçant une partie des immobilisations. Toutefois, les preuves d'efficience dynamique sont défavorables. Même lorsque le taux d'intérêt réel sur la dette publique est inférieur que le taux de croissance réelle de l'économie, les sommes consacrées à l'impôt et aux primes de risque

signifient que le rendement marginal du capital dépasse sans doute le taux de croissance réelle. Le coût réel de la dette publique correspond donc au rendement marginal de l'actif évincé en raison d'une dette publique plus élevée, et non au taux d'intérêt appliqué à la dette.

Dans une économie ouverte de faible envergure, la dette publique tend à influer sur l'actif étranger net plutôt que sur l'actif national. Le gouvernement peut cependant influer sur les taux d'intérêt au pays dans la mesure où les éléments d'actif ne sont pas des substituts parfaits à l'échelle internationale. Une augmentation de la dette publique accroîtra la dette extérieure nette, stimulant le rendement dont les investisseurs ont besoin pour soutenir la dette. Les considérations fiscales peuvent elles aussi engendrer un écart entre les taux d'intérêt au pays et ceux à l'étranger, tout comme les liens entre les taux d'inflation variables d'un pays à l'autre et les régime fiscaux non indexés. Enfin, si la demande d'exportations n'est pas parfaitement élastique, l'accroissement de la dette extérieure nécessite une détérioration des termes de l'échange par rapport à l'état stable. Le coût réel des emprunts du gouvernement dépasse donc le taux d'intérêt à l'échelle mondiale, quel que soit le taux de substitution de l'actif sur le plan international.

Qu'en est-il de l'avenir? Il semble que la libéralisation des marchés financiers et la faiblesse du taux d'inflation aient contribué de façon importante au relèvement des taux d'intérêt sur la dette publique, tout en facilitant l'accès des intervenants du secteur privé aux marchés de capitaux et en réduisant leurs coûts d'emprunt. Il est donc peu probable que l'écart entre les coûts d'emprunt des secteur public et privé revienne à leur niveau antérieur, à moins que l'inflation n'augmente ou que la libéralisation des marchés financiers ne régresse. Quant au niveau global des taux d'intérêt, ils pourraient subir des pressions à la hausse en raison de changements démographiques, d'une incapacité de contenir la croissance de la dette publique ou d'une combinaison de ces deux facteurs.

THE DETERMINATION OF REAL INTEREST RATES

Joel Fried and David Burgess

1 INTRODUCTION

This essay is concerned with the behavior of real interest rates in closed and open economies and in surveying recent developments in the theory of interest rate determination. Interest in this subject has been heightened in recent years because of the historically high rates of real interest on relatively riskless securities in the 1980's following a period of historically low rates in the 1970's. Governments in particular, are concerned, in part because of this increase in the cost of funds to finance their operations, in part because of the implications for economic welfare and, in part, because they require some guidance in formulating future policy.

To assist in understanding the movements in interest rates, and to provide a background for empirical work the government might wish to undertake to better formulate economic policy, this paper will address the following key questions: First, what are the fundamental determinants of real rates of interest and what determines their long run equilibrium values in the absence of shocks? Second, has there been a major change in the behavior of real interest rates and, if so, is this situation likely to persist, or can we expect real rates to return to levels more consistent with historical experience? Third, for an economy such as Canada's, to what extent are real rates determined by factors within a country and to what extent are they determined externally by factors beyond the country's control? Fourth, under what conditions can the after–tax real interest rate on government debt be less than the real economic growth rate, yet increased government debt imply a lower long–run level of economic welfare? Finally, what are the implications for government taxation, expenditure and debt policies of the recent behavior of real interest rates?

To examine these questions we begin, in Section 2 with a review of the neoclassical growth model. This provides an overall framework that can be used as a starting point to address the issue of the long run determinants of real interest rates, and to indicate the role of

government policies in influencing them. Section 3 surveys developments in the theory of consumption and saving under certainty and integrates the various economic models that have been used to examine the question of intertemporal choice. While there is some discussion of the effects of temporary shocks that will affect the rate of interest, sections 2 and 3 are primarily concerned with the first question, of the long run determinants of real interest rates.

Section 4 introduces uncertainty into our analysis. This allows us to examine interest rate differentials and to model issues of portfolio choice and of the term structure. While the existence of uncertainty modifies somewhat the level of interest rates in the long run, its basic contribution is in understanding shorter run movements in interest rates. This is particularly important in evaluating interest rate changes over the last two decades where, because of the oil shocks, it is reasonable to assume that there were shifts in the perceived level of economic uncertainty faced by individual agents.

Much of the existing literature on dynamic stochastic models presumes there is a complete set of contingent claims markets. However, the empirical literature suggests that these models fail to provide a fully satisfactory explanation of interest rate differentials. In Section 5 we therefore modify our model of interest rates to incorporate liquidity constraints and other aspects of incomplete markets. Again, this is important in understanding interest rate behavior over the last two decades in light of both the behavior of inflation and the changes that have occurred in the operations of financial intermediaries. We find that there is some reason to believe these factors played an important role in movements of real interest rates over this period.

The effects of government policies on interest rates are the major concerns of Sections 6 and 7. In the first of these we examine the effects of capital taxation, as well as providing some additional remarks on the investment decision of the firm. In Section 7 we examine the effects of other policies on the level of real interest rates as well as address the issue of dynamic inefficiency. This last issue bears on the question of whether there is a long run economic cost to the economy from issuing debt when the cost of funds to the government is

below the rate of real economic growth. Available evidence indicates that economies are not dynamically inefficient and therefore that there is an economic cost of debt issue, even when the cost of funds to the government is below the rate of output growth.

Section 8 is concerned with the influence an open economy has over domestic interest rates. We find that even under the extreme assumption that the economy cannot influence international interest rates or prices of goods, there is some role the government can play in affecting the cost of capital and the net return on the savings of domestic residents. We also consider the effects on domestic rates of interest when the economy is large enough to influence the international relative price of its exports.

In developing the model of interest rates in the body of the text we have, at times, become somewhat technical. We believe this is necessary if other researchers are to extend our work to examine some of the empirical issues raised. In addition we have attempted to relate each section to the key questions of this essay. This will appear at times to be a piecemeal approach to the issues. To provide an overall review of each of the questions we wish to address, as well as to make our analysis accessible to the non–technical reader we have, in Section 9, summarized our results for each of the five questions mentioned above and provided a brief verbal argument of the logic supporting them.

As we have carried out this project we have become increasingly aware of the limitations of current theories of interest rate determination and the paucity of data available to test hypotheses. The answers we have provided should therefore not be treated as the final statement of the theory of interest rates nor as complete description of them over the last twenty years. Rather, it is hoped that this review will stimulate some readers to further explore the behavior of interest rates, both empirically and from a theoretical perspective.

2 THE NEOCLASSICAL GROWTH MODEL

2.1 Introduction

In this section we review the basic neoclassical growth model.[1] The reason for doing so is to provide a common framework which can later be used to incorporate in summary form recent developments in modeling the behavior of economic agents. On its own it does not provide a satisfactory description of the determination of the array and behavior of interest rates we observe in the economy nor appropriate guidance to policy makers in formulating policy or understanding recent events. However, it does serve a useful purpose in pointing out the broad outlines of the determination of steady state interest rates and some of the effects of government debt policy on interest rates and economic welfare.

2.2 Basics

It is assumed that labor grows at the constant rate n so that at any date t, the supply of labor N_t, is

2.1) $N_t = N_0 e^{nt}$

Output, Y_t, consists of a single malleable composite good that can be consumed or converted to capital at a constant relative price normalized to 1. Output is produced using capital, K_t, and labor in a linear homogeneous technology:

2.2) $Y_t = F(K_t, N_t) = N_t f(k_t)$

where $k_t = K_t/N_t$ and $f(.)$ is the intensive form of the production function. Competitive markets are assumed so that capital owners and workers are paid their respective marginal products,

2.3) $r_t = f'(k_t)$, and

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[1]This model originated with Solow (1956). Modifications by Tobin (1965), Phelps (1961, 1966) and Levhari and Patinkin (1968) have also been incorporated. The analysis is well surveyed in Solow (1970).

2.4) $w_t = f(k_t) - f'(k_t)k_t$

where w is the real wage and r denotes the rate of interest which is here assumed to equal the rental rate on capital.[2] It is assumed that $f(k_t)$ is well behaved and that $f'(k) \geq 0, f''(k) \leq 0$, $f'(0) = \infty$, and $f'(\infty) = 0$.[3]

The government taxes all income (including interest on the government debt) at the rate τ and uses the proceeds from this tax to purchase goods of the amount G_t, provide net lump sum transfers to agents, Tr_t, and pay interest on the government debt, B_t. A unit of debt is normalized to equal one unit of consumption goods. The interest rate paid is competitively determined at the rate r_{bt} at date t and is paid in units of consumption goods[4]. The government faces the budget constraint:

$$\dot{B}_t = Tr_t + G_t + r_{bt}B_t - \tau(Y_t + r_{bt}B_t)$$

or, in intensive form,

2.5) $$\dot{b}_t = tr_t + g_t + (r_{bt}(1 - \tau) - n)b_t - \tau y_t,$$

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[2]Equation (2.3) is valid if a) there is no depreciation, b) there are no indirect taxes levied on the firm, and c) the price of capital is fixed in terms of consumption goods. If depreciation is assumed to be exponential, then interpret Y as net national product, $F(K_t, N_t) - \Delta^* K_t$ where Δ^* is the constant depreciation rate and $F(\cdot)$ is GNP. The analysis is the same as in the text. We discuss assumption (b) in section 6, and sections 3 and 6 address the issues raised by assumption (c).

[3]Various assumptions about technological change will modify the subsequent analysis somewhat. However, the principal assumption that is usually made is that change is Harrod neutral. Letting ν be the constant rate of Harrod neutral technical change, a worker at date t is the equivalent of $e^{\nu t}$ workers at $t = 0$. Thus, in terms of <u>effective</u> labor units at date t, N_t^*,

$$N_t^* = N_0 e^{(n+\nu)t}$$

Thus k_t can be interpreted as the <u>effective</u> capital labor ratio, K_t/N_t^*, and the analysis proceeds with N_t^* in place of N_t.

[4]The similarity of this model to the money and growth literature (eg. Tobin (1965), Levhari and Patinkin (1967)) should be noted. In that literature money is simply government debt when debt servicing is identically equal to zero. Because our concern is with steady states, it is more convenient to assume the existence of real valued debt and to ignore the capital gains that arise from "capital levies" in the form of changes in the price level.

where tr, g, b, and y are the per worker levels of Tr, G, B and Y, and $\dot{x} = dx/dt$, the time derivative of x. Finally, g_t can be viewed either as consumption goods wasted (e.g. dumped into the sea) or as government purchases of capital goods.[5] If it is the latter, any revenue generated by the outstanding stock of government capital goods would enter negatively on the RHS of (2.5). For expositional purposes we shall suppose that $g_t = 0$ for all t. In this case per worker transfers become

2.6) $$tr_t = \dot{b}_t + (n - (1 - \tau)r_{bt})b_t - \tau y_t$$

In this environment agents have two decisions to make: first, what proportion of their disposable income will be saved; and second, how will their tangible wealth be allocated between private capital and government debt. For the former, it is often argued that savings depends upon the agent's level of wealth, V_t, and on the vector of real, after tax interest rates. Given the inter–relationship between income, wealth and interest rates, this implies that savings also depends upon some measure of current and/or expected future disposable income. The assumption used in this section will be the Keynesian one, that savings is a constant fraction, s, of current measured disposable income:

2.7) $$S_t/N_t = sy_{dt} \equiv s((y_t + r_{bt}b_t)(1 - \tau) + tr_t)$$

where S is total savings and y_d is disposable income per worker.

For the portfolio decision it is assumed that the ratio of government debt demanded relative to capital depends on the relative yields on debt and capital. That is:

2.8) $$b_t = \ell(r_{bt}, r_t)k_t,$$

where $\partial\ell/\partial r_b > 0 > \partial\ell/\partial r$.

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[5]The existence of public goods raises the problem of how to define consumption expenditures. Therefore, public goods are ignored here for expository purposes. If government goods are not public in nature and are not "wasted", then they are accounted for as transfers.

Equilibrium requires that the demand for new assets by households equal the supply of new assets by firms and the government, and that assets be allocated according to (2.8). The per worker supply of assets is

2.9) $I_t/N_t \equiv \dot{K}_t/N_t + \dot{B}_t/N_t \equiv \dot{k}_t + \dot{b}_t + nk_t + nb_t$

and the demand for these assets is given by (2.7). Equating (2.7) and (2.9) gives

2.10) $nk_t + \dot{k}_t = sy_t - (1-s)(b_t n + \dot{b}_t)$.

The steady state requires $\dot{k} = \dot{b} = 0$. Substituting this and (2.8) into (2.10) provides the steady-state equilibrium condition,

$$n\bar{k} = s\bar{y} - (1-s)n\ell(\bar{r}_b, \bar{r})\bar{k},$$

or

2.11) $\bar{k} = \{s/n(1 + (1-s)\ell(\bar{r}_b, \bar{r}))\}\bar{y}$

where the overbar denotes steady–state values. Thus the steady state capital stock depends positively on the savings rate s, and negatively on the stock of government bonds outstanding. The quantity of these bonds that are willingly held is, in turn, positively related to the rate of return on bonds relative to the yield on capital.

2.3 Government Debt Policies

One of the major insights from the neoclassical growth model is the concept of dynamic inefficiency, whereby the outcome of individual savings decisions need not be Pareto efficient. Dynamic inefficiency occurs when the marginal product of capital is less than the (effective) rate of population growth so the capital stock is greater than its golden rule level[6].

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[6]The golden rule level of the capital stock is that steady–state capital stock that maximizes consumption and occurs when $f'(\bar{k}) = n$: Differentiating $\bar{c} = f(\bar{k}) - n\bar{k}$ w.r.t. $\bar{k}$ gives $f'(\bar{k}) = r = n$ for maximum consumption.

If the economy is dynamically inefficient, a permanent reduction in the capital stock can increase the consumption levels of some agents without reducing the consumption of any other agent. If holding capital per se, does not generate utility while consumption does, a decrease in the capital stock must be Pareto improving in a dynamically inefficient equilibrium. One of the roles for government debt is to ensure that the economy does not end up in a dynamically inefficient state. It does this by "crowding out" some of the "excess" capital stock.

To examine the effect of government debt on the steady state capital stock, consider the following two rules for the increase in government debt: (i) the government fixes the steady state ratio of b/k at ϕ, and (ii) the government fixes the steady state r_b at $\bar{r}_b$.
In case (i) equation (2.11) becomes:

$$\bar{k} = \{s/n(1 + (1-s)\phi)\}\bar{y}$$

and $\partial\bar{k}/\partial\phi < 0$ which describes the traditional crowding out exercise of the undergraduate macroeconomics class. It should also be noted that there must be a steady state budget deficit of the amount $n\phi k^*$. Insistence on a budget balance implies that $\lim_{t\to\infty} \phi = 0$.[7]

Case (ii) is qualitatively the same as that discussed in the early literature on the optimum quantity of money. Fixing $\bar{r}_b$ is identical to fixing the rate of inflation and, in the steady state, the rate of monetary growth. Under these conditions (2.11) becomes:

$$\bar{k} = \{s/n(1 + (1 - s)\ell(\bar{r}_b, \bar{r}))\}\bar{y}$$

and $d\bar{k}/d\bar{r}_b < 0$: Increases in $\bar{r}_b$ cause the demand for government debt to increase, crowding out capital.

Given the arbitrary savings assumption made, it is possible that an economy could find itself in a dynamically inefficient equilibrium. How can one identify when such a circumstance exists? One method is to compare net capital investment, nk, in the steady

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[7]If transfers are not taxed there will be a limit to the amount of government debt the economy is capable of absorbing for any minimum positive value of $G + Tr$—a sort of "Laffer curve" for debt—since, at an arbitrarily large ϕ no output will be produced as the capital stock (and b) will approach zero.

state, with capital's share of output, rk in equilibrium. If the former is larger than the latter, or alternatively, the capital goods sector is a net demander of resources (i.e. a sink ranther than a spout in the words of Abel, et al (1989)), then the equilibrium is dynamically inefficient.[8]

A second method of identifying dynamic inefficiency is to compare directly the marginal product of capital net of depreciation with the effective growth rate of the labor force. There are at least two measurement problems here. First, due to technical change, the effective labor force growth may be difficult to measure. A solution to that is to use the growth rate of output itself, which will coincide with that rate in the steady state. The second problem is in the choice of interest rate to compare with the growth of output. In the model so far developed, both r and r_b vie for contention. r, however, is the appropriate choice since the concern is with commodities available for consumption instead of capital. Reducing the capital stock by one unit indefinitely provides households with n additional units of consumption each period at the cost of foregoing r units of consumption each period and the basic calculus of choice is obvious. The use of r_b has no such immediate and obvious an interpretation. Indeed, one of the obvious inadequacies of the neoclassical growth model is that there is no apparent reason why this rate should differ from r.

Drawing on the money and growth literature, however, does offer a possible interpretation of any difference that might exist: The government debt provides some form of (unspecified) non–pecuniary services (Levhari–Patinkin, (1968)). Incorporating this into the neoclassical growth model would require modification of disposable income to incorporate the value of these services, but would not lead to major changes in describing the steady state. It would, however, provide some insight into what the optimal level of government debt might be. Suppose the government is concerned with maximizing the steady state level of utility and has only the budget deficit as its control variable. If debt provides services of some sort, then

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[8]Cf. Abel, Mankiw, Summers and Zeckhauser (1989). On this criterion the North American economies clearly do not appear dynamically inefficient.

the "instantaneous" utility function of the representative agent in the steady state can be characterized by:

$$u(\bar{c},\bar{b}) = u(f(\bar{k}) - n\bar{k}, \bar{b}).$$

Differentiating this totally with respect to $\bar{b}$ and setting the result equal to 0 for a maximum gives:

$$du/d\bar{b} = u_1(f'(\bar{k}) - n)d\bar{k}/d\bar{b} + u_2 = 0.$$

If $r_b < r$, then u_2 is positive. Non–satiety of consumption means that u_1 is positive as well. Further, because $d\bar{k}/d\bar{b}$ is negative, for an optimum, $f'(\bar{k})$ should thus be greater than the effective rate of labor growth. Unfortunately, the model does not provide much guidance as to what the appropriate level of debt should be.

At this point it should also be noted that the assumption of only one instrument to achieve the optimum is restrictive. Suppose, for instance, that government spending can be used to purchase private capital and/or public goods. Then the government has sufficient tools to choose both $\bar{k}$ and $\bar{b}$. In that case the optimum would be characterized by $r = n$ and u_2 set at its optimal level. That optimum will be such that the marginal utility of debt will equal the marginal cost of maintaining the debt at its optimal level.

2.4 The Open Economy

One assumption that plays an important role in the above model is that the supply of savings for capital accumulation is completely inelastic with respect to the rate of interest. It is also useful to explore the model under the alternative assumption that the supply of savings is quite elastic with respect to the interest rate. Rather than examine household savings behavior in detail, suppose instead that the economy we are examining is a small open economy and that there is perfect capital mobility. As well, treat all assets as perfect substitutes.

Under these assumptions the supply of funds for capital projects in the home country is infinitely elastic at the given (after tax) world rate of interest, r_w. The government can manipulate the marginal product of capital by altering the tax rate on capital provided that it has the ability to alter the marginal tax rate on all income generated in its territory. Thus, the marginal product of capital, and rate of interest observed at home, is:

$$r = r_w/(1 - \tau).$$

If $r_w > n$, the world (and home country) population growth rate, the home country is clearly in a dynamically efficient equilibrium. The interesting question is what happens if r_w is less than n and, if so, whether an increase in the tax rate or an increase in government debt is the appropriate policy. First of all, in this economy government debt, per se does not crowd out domestic capital. Thus debt policy can be viewed independently of that issue. As a consequence, if $r < n$, government is advised to crank up the debt for, in the steady state, new debt issue will cover the interest payments on the old debt plus provide a surplus to the current citizens of the country, and further, this surplus will, at the margin, be provided by non–residents. This policy of course would be followed by all governments, thus removing the opportunity. What about tax policy? Would a tax rate, $\tau = 1 - r_w/n$, to bring about $r = n$ improve welfare? The answer appears to be no if the government is concerned only with the welfare of its own citizens. At the margin, foreigners are providing the capital and in so doing, increasing the marginal product of resident workers. Thus foreigners are indirectly subsidizing domestic residents and there is no reason the government should try to kill the golden goose.

2.5 Summary

The neoclassical growth model presented above is useful as a starting point in examining the rate of interest and the effect of governments on it. It does contain in broad outline those forces determining the rate of interest in the long run. The rate of interest is ultimately determined by attitudes toward saving and the technology of production.

Government influence on that rate is two fold. First, by providing an alternate vehicle for transferring purchasing power over time, a wedge is driven between the quantity of funds provided and the quantity allocated to increasing the capital stock. Secondly, through tax policy, a wedge is driven between the price at which firms obtain funds and the effective price agents receive for allocating funds to future consumption. This is most easily seen in the small open economy case, but it would also hold for the world economy if household savings behavior was sensitive to wealth and/or interest rate levels. Furthermore, it does identify a role for government debt in overcoming dynamic inefficiency, when the marginal product is less than the rate of population growth. However, beyond these broad conclusions the model is seriously deficient in providing guidance to explain a number of observable phenomena in on–going economies. Three such observations that need to be explained are:

1) Why do we observe shifts in the savings rate? Does the level of savings depend upon variables other than disposable income, and if so, what are these variables and how are they influenced by the government?

2) Why do we observe such a wide variety of different stores of value having different intertemporal relative prices?

3) Why do interest rates fluctuate to the extent that they do? Ie, what are the transitional properties of interest rates and, indeed, the economy in general?

Examining these three questions will give us some insight into the behavior of interest rates and, as well, clarify what are appropriate government policies in light of the principles explaining this behavior.

We shall address these issues in the following sections of this essay. All three require a greater focus on the microfoundations of the macroeconomy. Briefly, these considerations require us to be more specific about the choice problem confronting agents and firms, and the economic environment in which they find themselves. That environment has two basic aspects that are not explicitly incorporated into the neoclassical model: Uncertainty regarding the

states of nature through which the economy will pass in the future, and the absence of a complete set of Arrow–Debreu markets for all goods in all states of nature of the economy. We designate these two elements as risk and liquidity respectively. The next three sections examine advances in modeling household behavior that have been developed to address these three issues for a closed economy. In subsequent sections we examine firms and government to see how their structure will influence household behavior and, through their actions, influence interest rates.

3 THRIFT AND THE RATE OF INTEREST

3.1 Introduction

In this chapter we analyze thrift and the rate of interest to more fully specify the savings function used in the neoclassical growth model of the previous section. The starting point is the modern theories of the consumption function begun with the seminal contribution of Friedman (1957) on the Permanent Income Hypothesis and by Modigliani and Brumberg (1954) on the Life Cycle Hypothesis.[1] These theories take as their point of departure that consumption decisions are based on private maximizing behavior and have been more careful than in the past in specifying both the maximization problem facing the agent and the economic environment in which the agent operates. The result of this changed emphasis is that personal saving—thrift—is viewed more as a residual between disposable income and a forward looking consumption function that depends upon the agent's tangible and intangible wealth and upon relative intertemporal prices—the rates of interest. One of the aims of the present chapter, then, is to explore just how far this alternative view of savings affects qualitatively the implications of the neoclassical growth model for the determination of interest rates.

In being precise about the maximization problem that the agent faces economists have used a wide variety of specific assumptions covering, among other things, the length of the planning horizon, the nature and extent of the assets that can be held, the degree of certainty involved in the decision and the nature of the exogenous forces impinging on the agent and his environment. In part this is because the specific problem the economist wished to address was more amenable to solution using one set of assumptions instead of an alternative set. This can occasionally leave the reader with the view that implications for interest rates are entirely

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[1] A large and growing literature has now developed on extensions of this original work. For a sampling c.f. Campbell and Mankiw (1989), Cristiano (1987 a, b), Deaton (1987), Dolde (1980), Flavin (1981), Hall (1978, 1982, 1988), Mankiw and Shapiro (1985), Michner (1984), Modigliani (1986), Skinner (1988) and Zeldes (1989 a, b).

assumption specific or, at the very least, leave him with a splitting headache. The function of this and the following two chapters is therefore to sort out how germane these assumptions might be for the determination of the rate of interest. We cannot, of course, cover the entire spectrum of assumptions made, so we shall instead concentrate on just a few of the major modeling assumptions.

One of the principal modeling differences in the literature is that of length of the planning period, some authors supposing an infinite horizon, others specifying a finite horizon of precisely two periods. We shall show that, with the exception of certain specialized cases, the horizon assumption is immaterial to many of the major propositions, such as dynamic efficiency and debt neutrality that are central to the theory of interest. Instead, it is the subsidiary assumptions relating to income streams, heterogeneous agents, distribution effects, and the like that lead to different conclusions in the two models rather than the finiteness of the individual planning horizon per se.

A second assumption that appears to differentiate the product of authors is the nature of the assets held, some supposing capital is reproducible, others supposing it is not. We model both cases, not because they are fundamentally different in their long run implications—which they are not—but because the behavior in the transition encompasses the range of responses one would obtain in more realistic models where both asset prices and quantities adjust.

The third major assumption we choose to examine is the modeling of uncertainty. We begin with an environment where agents have perfect foresight, primarily because of its familiarity, and only later, in Chapter 4, do we consider the introduction of risk. While the uncertain world is richer in its implications, the environment of perfect foresight does permit the establishment of certain basic principles that carry over to the world of risk.

The final modeling assumption we choose to address is that of perfect capital markets. This assumption, which is common to almost all of the formal literature on asset pricing is powerful as an analytical tool yet some would express doubt as to its realism. In chapter 5 we

discuss some of the qualifications that appear to arise if this assumption is violated.

We begin with a statement of the individual experiment under certainty where we note the similarity of the decision problem under different planning horizons. Following that we examine the market experiment for the infinite lived representative agent when the only asset of choice is reproducible capital. The analysis is then replicated when land is the only asset. The land case permits us to conveniently model the effects of temporary and permanent shocks to the economy on the demand for capital. In this case, the shock manifests itself on the price of assets which, we argue later, in section 6, will map into the rate of change in the capital stock when capital goods can be accumulated. Some modifications to the basic choice model are then examined in the overlapping generations model. We conclude the section with a summary.

3.2 Basics

Assume that agents have perfect foresight and there are no unanticipated shocks to the economy. The agent's decision problem is generally formulated as one where preferences are additive and time separable in consumption goods. In addition, a constant rate of time preference, δ, is most often assumed. These agents wish to maximize

3.1) $$\phi_0 = \sum_{t=0}^{T} \beta^t u(c_t)$$

subject to

3.2) $$a_{t+1} = R_t(a_t + \omega_t - c_t), \quad 0 \le t \le T,$$

where

T is the "planning horizon" for the agent,

$\beta = 1/(1 + \delta)$,

ω_t = endowment, or non–asset, income in period t,

$R_t = 1 + r_t$, the gross return on a unit of assets at date t,

r_t is the one period rate of interest, and

a_t is the value of assets measured in units of consumption goods held at the end of period t.

To rule out Ponzi games and unbounded borrowing, it is assumed that, for all $t \geq 0$,

$$3.3) \quad c_t + \sum_{j=1}^{T} \left(\prod_{0}^{j-1} R_{t+k}^{-1} \right) c_{t+j} \leq \left[\omega_t + \sum_{j=1}^{T} \left(\prod_{0}^{j-1} R_{t+k}^{-1} \right) \omega_{t+j} \right] + a_t = h_t + a_t \equiv V_t$$

where h_t is the present value of non–asset income streams, ω_t. This income may accrue in the form of labor income, net government transfers, etc. Current holding of tangible assets, a_t, plus h_t constitutes the total wealth, V_t, of the agent and represents the ultimate constraint on the agent's consumption decision. The distinction between h_t and a_t is a useful construct for some purposes. First, it forces the economist to identify all income streams accruing to the agent, whether or not there is a specific market on which titles to those income streams trade. Second, it brings to the forefront questions about the conditions necessary for h_t and a_t to be treated as perfect substitutes. If there are a complete set of Arrow–Debreu markets, then the conditions are met; if some of these markets do not exist, then they might not be and the economist is lead to focus on the cause of this market failure.

If condition (3.3) holds then the maximization problem can be restated as maximize (3.1) subject to (3.3). Solving this problem gives the consumption function,

$$c_t = c(V_t, R_t)$$

where $\partial c/\partial V_t > 0$ and $\partial c/\partial R_t \lesseqgtr 0$.[2] In the interest of specificity we shall make use of the

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[2]It should be noted that $\partial c_t/\partial R_t$ is conditional on a given V_t. If, in fact, $\omega_{t+1},\ldots,\omega_T \geq 0$ are exogenous and some are positive, then $\partial h_t/\partial R_t < 0$ and a change in R_t will change both arguments in the consumption function making it less likely that $dc_t/dR_t > 0$. These "wealth effects" are sometimes finessed in the literature by supposing endowment income is only received in the first period of an agent's life.

constant relative risk aversion (CRRA) preference function[3]

$$u(c_t) = \begin{cases} c_t^{(1-\gamma)}/(1-\gamma) & 1 \neq \gamma \geq 0 \\ \log\ c_t & \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion,

$$\gamma = -c_t u''(c_t)/u'(c_t)$$

While such preciseness of preferences may detract from possible generalizations that can be made, the class is sufficiently broad to encompass a wide variety of behavior, and in particular provides a simple means of identifying two parameters that have interested researchers, vis γ and the elasticity of intertemporal substitution, σ. σ is a measure of the responsiveness of consumption to changes in the rate of interest and equals the inverse of γ. In the case of CRRA preferences, $\partial c_t/\partial R_t \gtrless 0$ as $\gamma \gtrless 1$. In the case of $\gamma > 1$ agents have a relatively strong preference for consumption smoothing so that, for given V_t, the increased future income is "smoothed" by increasing both current and future consumption.

Because measured saving is the residual between current income and $c(V,R)$, the notion of a constant savings rate to tie down the steady state capital stock is militated against. In particular, a wealth compensated change in current income (such as an increased tax now instead of in the future) will alter current income without altering consumption and will imply a marginal propensity to save of 1. On the other hand changes in R that accompany economic growth will also alter savings rates and, if $\partial c_t/\partial r_t > 0$, could imply a negative relationship between income and savings rates. The decision rule for consumption is given by the Euler equation of the maximization problem:

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[3]This class of preferences is also called constant elasticity of substitution (CES) preferences because they have a constant elasticity of intertemporal substitution, $\sigma = 1/\gamma$. We use the term CRRA for consistency because we will be introducing risk in section 4. Note that CRRA preferences has the convenient property of generating consumption demand that is homogeneous of degree one in wealth so that consistent aggregation can be invoked. It is also amenable to closed form solutions in stochastic environments so it is often used in empirical work on real business cycle theory. It does not, however, appear to be especially consistent with aggregate data. C.f Hansen and Singleton (1983).

3.4) $u'(c_t) = \beta R_t u'(c_{t+1})$

The LHS of (3.4) indicates the cost of forgoing one more unit of consumption now and the RHS shows the gain in future utility if current consumption is reduced by that amount. It is (3.4) that governs the behavior of consumption, not disposable income.[4] Thus, for the individual agent, the rate of interest relative to the rate of time preference will provide the link between trend income and consumption: if $r > \delta$, the agent will, on average, be accumulating assets and thus be choosing both a higher level of income, and , from (3.4), a higher level of consumption. In the case of CRRA preferences, (3.4) becomes

$$\beta R_t = (c_t/c_{t+1})^{-\sigma}.$$

The greater is σ (or lower is γ) the less will R have to change to evoke a given change in consumption behavior.

Our interest is in characterizing the steady, or stationary, state. One approach is to suppose the existence of an infinite lived representative consumer whose behavior is characterized by (3.4) with $T = \infty$, and who receives the representative endowment, etc. Then the description of his behavior serves to describe the levels of the per capita averages for the quantities of interest to the economist. This is done in the dynastic agent paradigm.

An alternative approach is the overlapping generations (OG) paradigm with T set at 2. Here we can also describe a representative agent in any period, and agents across periods, as having identical preferences. However, at any given date there are two agents alive who differ in age, and face different constraints and endowments. They therefore cannot be treated as if they had a common set of preferences and constraints at any given date. Instead, to describe the steady state we require that the economy replicate itself from period to period with the old of generation $t + 1$ behaving as did the old of generation t and the young of the t^{th} generation

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[4]See Hall (1978, 1988), or Flavin (1981) for empirical evidence on the use of (3.4) in estimating the consumption function. See also Michner (1984). See Hayashi (1987) for a discussion of the use of (3.4) rather than the consumption function for empirical work.

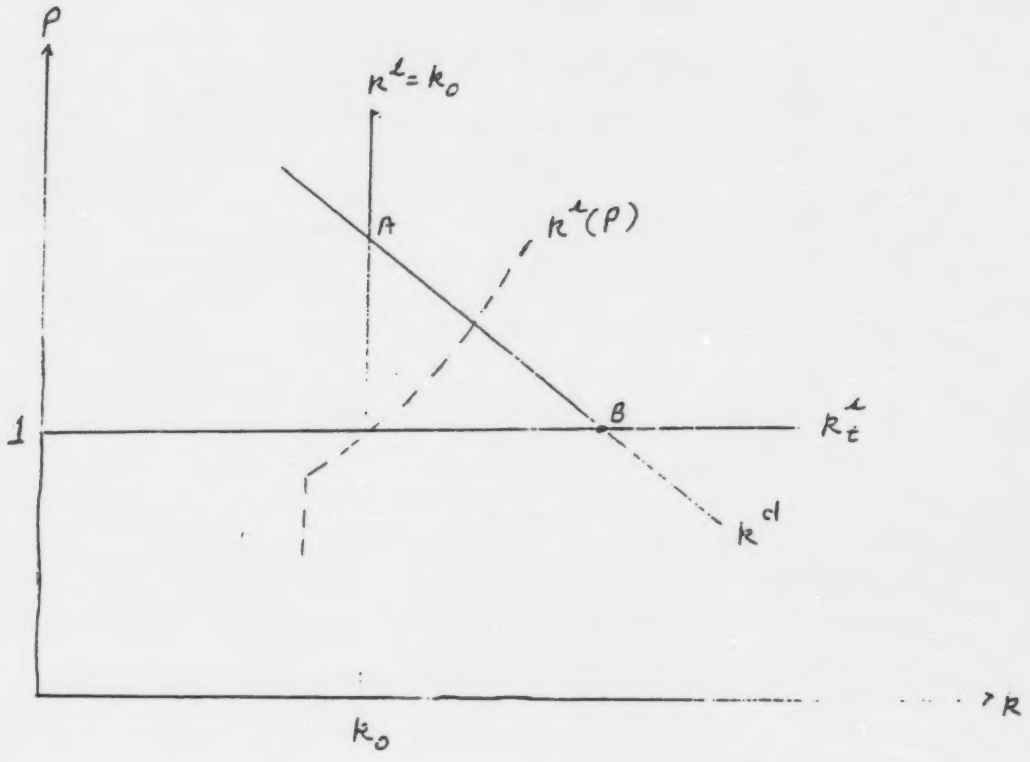
behaving as did those of period $t-1$.

To determine the behavior of interest rates it is also necessary to provide some description of the economic environment in order to establish the type of outcomes that are possible. We consider two limiting economic environments. In case 1, designated the capital model, a factor of production, capital, is reproducible and the relative price of capital goods in terms of consumption goods is fixed and normalized at unity. Here the quantity of reproducible capital adjusts to equate demand and supply over time and, in the process, causes the rate of interest to adjust. In case 2, the land model, it is assumed that factors of production are fixed in supply. Thus the only mechanism that can be used to bring about an equilibrium are prices and, in particular, the price of assets. The reason we examine these two cases is that they "bracket" the set of realistic cases we can expect where a change in the savings behavior may affect both the price of capital goods and the quantity of capital in the short run. This is sketched in figure 3.1. The reproducible capital case supposes the horizontal supply curve for capital goods whereas the land model posits a vertical supply schedule. In a two sector model, or in one where there are costs of adjusting the capital stock, there will be a positively sloped capital goods supply schedule intermediate to these two schedules and intersecting the demand schedule somewhere between points A and B, given an initial capital stock k_0. What we explore in this chapter is the nature of the demand for capital schedule and its affect on the rate of interest. Our assumptions about land and reproducible capital permit us to do so looking only at quantity adjustment and then only at price adjustment.

3.3 Dynastic Agents and the Capital Model

We begin by examining the case where the decision unit is infinite lived and has the opportunity to alter the capital stock by investing in capital goods that trade at a unit price

FIGURE 3.1



The demand and supply of capital in the one sector model and the land model

with consumption goods.[5] The decision unit can be viewed as a family that grows at the rate n and is intergenerationally linked through bequests and a certain amount of intergenerational benevolence. We suppose that all families have the same preferences and members of any family have the same preferences across generations. The production technology is the same as that used in the neoclassical growth model given by (2.2)–(2.4).

In describing the determination of the rate of interest in this model suppose time is continuous. Then the decision problem is to maximize

3.5) $$\phi_0 = \int_0^{\infty} u(c_t)\, e^{-\delta t} dt$$

subject to the law of asset accumulation of

3.6) $$\dot{a}_t = \omega_t + (r_t - n)a_t - c_t$$

where $\dot{x} = dx/dt$, the time derivative of x. In the absence of government, let $a_t = k_t$, $\omega_t = w_t$ and k_0 given. Forming the Hamiltonian,

$$H_t = e^{-\delta t}[u(c_t) + \lambda_t(\omega_t + (r_t - n)k_t - c_t)]$$

and maximizing, gives the condition (3.6) and the Euler condition,

3.7) $$\dot{\lambda}_t/\lambda_t = -[r_t - n - \delta]$$

to describe the evolution of a_t and λ_t, where λ_t is the shadow price of capital. Along the optimal path,

3.8) $$u'_t = \lambda_t.$$

Using (3.8) and noting that $\dot{u}'_t = u''\dot{c}_t$, (3.7) can be rewritten as

3.9) $$\dot{c}_t/c_t = -(u'/c_t u'')[r_t - n - \delta] = \sigma_t(r_t - n - \delta),$$

where σ_t is the elasticity of substitution of consumption.

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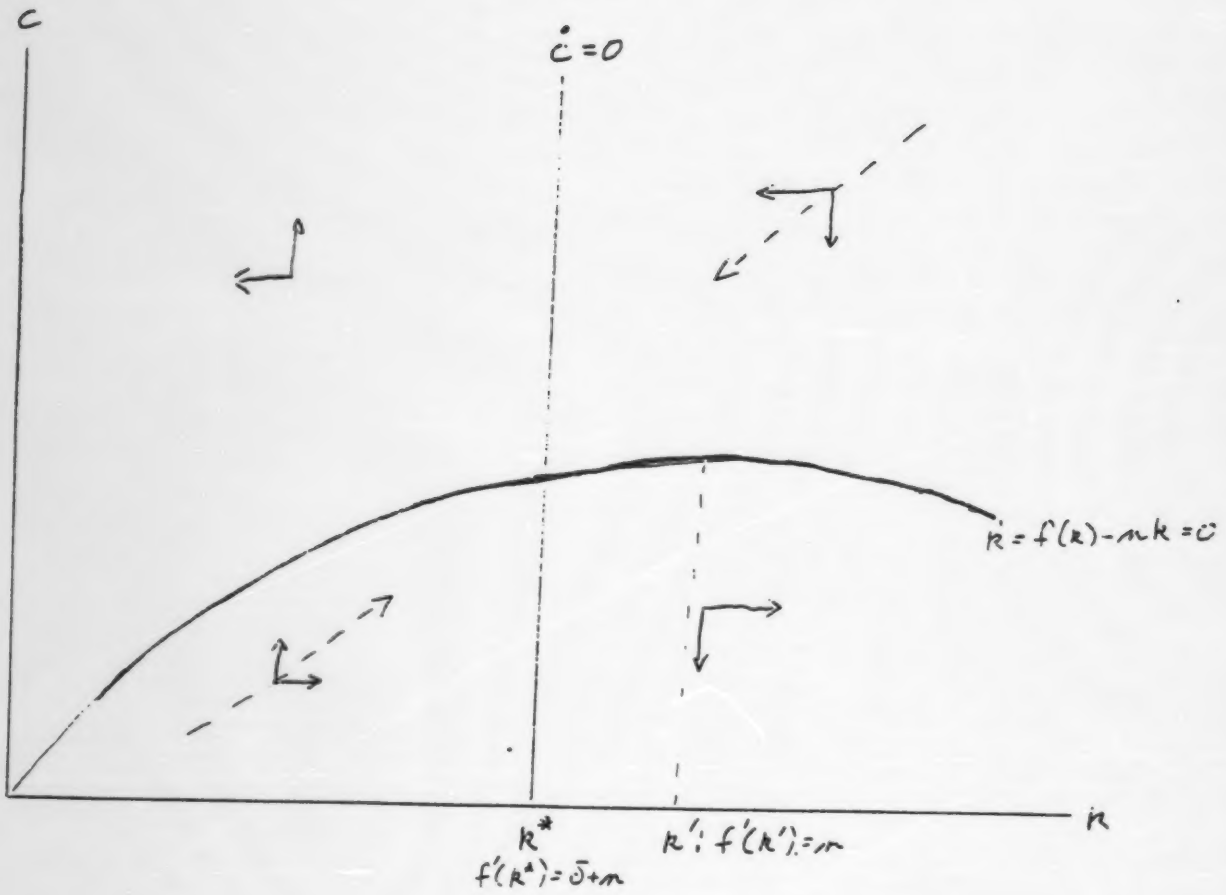
[5]This model's origins can be traced to Ramsey (1928) although he dealt with the central planning problem for optimal growth. In that framework the objective function is (3.5) below with an effective discount rate of $\delta - n$ and a steady state equilibrium of $f'(k) = r = \delta$.

FIGURE 3.2



Equilibrium in the dynastic agent model

The economy is described by the phase diagram in Figure 3.2. The $\dot{k} = 0$ and $\dot{c} = 0$ curves correspond to (3.6) and (3.9) set equal to zero. The steady state equilibrium occurs at the intersection of these two curves and generates at equilibrium rate of interest,

3.10) $\quad r^* = f'(k^*) = \delta + n.$

Thus, given the exogenous rate of population growth, the steady state interest rate depends only on the decision unit's rate of time preference. This follows from (3.4). If population growth is zero then, if consumption is to be constant, $\beta R = 1$, or $r = \delta$. Essentially, if there is sufficient time to accumulate assets one can choose any income level one wishes. In this case the agent accumulates to the point where the value of an additional unit of capital held indefinitely is not worth even the temporary loss of consumption for one period. The reason for $r > \delta$ in the growth case is that agents accumulate assets for the new members of society. This can only be accomplished if the incentive to save $-r > \delta -$ is in place. Note that, from (3.10) this means that the economy will never be dynamically inefficient. Also note that, from (3.9), the elasticity of substitution may influence the speed at which the economy approaches the steady state, but does not affect the steady state equilibrium itself.[6]

3.4 The Land Model Under Certainty

The basic model of the previous section is especially useful in analyzing the steady state rate of interest where the price of capital goods in terms of consumption goods will be constant. In the transition however, it may be that this relative price is changing. To provide a richer description of the determination of interest rates out of the steady state we find it convenient to suppose that the capital stock is fixed and that the price of capital, P_t, adjusts to clear commodity and asset markets. To distinguish this conceptual framework from the

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[6]Suppose that instead of population growth there was Harrod neutral technological change at the rate ν. In this case, (3.9) would be described by $\dot{c}/c = \sigma_t(r_t - \nu - \delta) = \nu$ in the steady state. In this case, if the steady state exists, $r_t = \delta + \nu(1 + 1/\sigma_t)$. A necessary condition for a steady state is that σ_t be constant, i.e. preferences are CRRA.

standard case we designate the capital stock as land, K^{ℓ}.

To examine the determination of the rate of interest in the land model it is convenient to suppose the supply of labor is constant at $N_0 = 1$ and that there is a third "factor of production", weather, that can make the stock of land more or less productive.[7] Describe total production at any date t by

$$Y_t = F(N_t, K_t^{\ell}, \varepsilon_t) = N_t(f(k_t^{\ell}) + \varepsilon_t k_t^{\ell})$$

where $k_t^{\ell} = K_t^{\ell}/N_t = k^{\ell}$ and $\varepsilon_t k^{\ell}$ is the effect of weather on output per worker.

$$\varepsilon_t \in [\varepsilon^-, \varepsilon^+], -f'(k^{\ell}) \leq \varepsilon^- < 0 < \varepsilon^+.$$[8]

Competitive factor markets imply that the rental rate on land and the real wage are, respectively:

$$\theta_t = f'(k_t^{\ell}) + \varepsilon_t$$

$$w_t = f(k_t^{\ell}) - (f'(k_t^{\ell}) + \varepsilon_t)k^{\ell}$$

With no government, let ω_t in (3.2) equal w_t.

The gross rate of return on a unit of land at date t is

3.11) $$R_t = (P_{t+1} + \theta_{t+1})/P_t$$

where P_t is the (ex dividend) price of a unit of land at date t. The present value of a unit of land is,

3.12) $$P_t = (P_{t+1} + \theta_{t+1})/R_t = (P_{t+2} + \theta_{t+2})/R_t R_{t+1} + \theta_t/R_t = \ldots$$
$$= \sum_{j=1}^{\infty} (\prod_{0}^{j-1} R_{t+k}^{-1})\theta_{t+j}$$

---

[7]It should be noted that there is an internal inconsistency in conducting these types of comparative static experiments in a certain world. The present analysis can perhaps be justified as an approximate treatment of what would happen if surprise weather changes were very rare events, in which case there would not be much uncertainty in the economy. Alternatively, we do point out the consequences if the weather change had been anticipated at an earlier date, j, that could be the beginning of time.

[8]With unlimited liability and the assumptions we have made the lower bound on ε_t could be as low as $-f(k^{\ell})/k^{\ell}$ but there is no reason at this stage to consider this sort of eventuality.

In the case where P_t is constant, as it will be in the stationary state, and θ is constant, (3.12) becomes $P_t = P = \theta/r$.

First it is easily confirmed that the model is consistent with the dynastic agent capital model. Suppose $\varepsilon_t = 0 \; \forall \; t$. Then real income will be constant at $y = f(k^{\ell})$, $\theta = f'(k^{\ell})$, and the goods market clearing condition will be $c_t = y \; \forall \; t$. From (3.4) $R_t = u'(y)/\beta u'(y)$ is constant at R, and, since θ is constant, using (3.12) so too is P_t at

3.13) $$P = P_t = \beta(P_{t+1} + \theta)[\psi(y_{t+1}, y_t]$$

where $\psi(a,b) = u'(a)/u'(b)$. Solving for P_t when $P_t = \bar{P}$ and $y_t = \bar{y} \; \forall t$ gives $\bar{P} = \theta\beta/1-\beta$. Using (3.11), this gives $r = \theta/P = (1-\beta)/\beta = \delta$. Further, an exogenous increase in land causes θ to fall, decreasing P. If the cost of bringing more land into production in terms of consumption goods is one then we are back to the reproducible capital case of the previous section.

Of more interest in the land model is the effect of a transitory or permanent shift in the weather factor ε. To examine these issues suppose, as our point of reference, that $\varepsilon_t = 0 \; \forall_t$. This generates an asset price $\bar{P}$, income level $\bar{y}$, and interest rate $\bar{R}$. Then, for a transitory shock at date $t = 1$, there is an unanticipated increase in ε to $\underline{\varepsilon} > 0$ for period 1, all other ε_t remaining at zero. For a permanent shock suppose that $\varepsilon_t = \underline{\varepsilon} > 0$ for all $t \geq 1$. Consider the transitory shock first.

At $t=1$ we know that $P_t = \bar{P} \; \forall \; t > 1$. Thus the rate of interest at $t=1$ is

3.14) $$R_1 = (\bar{P} + \bar{\theta})/P_1 = u'(\bar{y} + \underline{\varepsilon}k^{\ell})/\beta u'(\bar{y}) < \bar{R}.$$

It follows immediately that the asset price in period $t=1$ is greater than $\bar{P}$ which was the prevailing price before and after $t=1$.

What if, at $t=0$, the increase in θ_1 was anticipated? Then:

3.15) $$R_0 = (P_1 + \theta_1)/P_0 = u'(\bar{y})/\beta u'(\bar{y} + \underline{\varepsilon}k^{\ell}) > \bar{R}.$$

Because $\theta_1 > \bar{\theta}$ by assumption, and $P_1 > \bar{P}$ from (3.14) it is possible that (3.15) could hold at

$P_0 = \bar{P}$ but this is unlikely. Indeed, the expected increase in the "dividend", θ in period 1 could cause the price of a title to that dividend to increase or decrease.[9] Furthermore, it can be shown that if the increase in θ_1 is first anticipated in period $j < 0$, then $\bar{P} < P_j < P_{j+1} < ...P_0$ if $P_0 > \bar{P}$ and $\bar{P} > P_j > P_{j+1} > ...P_0$ if $P_0 < \bar{P}$.[10] That is asset prices approach P_0 monotonicly from the point at which the change in θ_1 is first anticipated.

A permanent increase in ε_t for all $t \geq 1$ will leave the rate of interest unchanged for all $t \geq 1$ since $R_t\beta = u(\bar{y} + \underline{\varepsilon}_t k^{\ell})/u'(\bar{y} + \underline{\varepsilon}_t k^{\ell}) = 1$, $t \geq 1$ but since $\theta_t > \bar{\theta}$, this means $P_t \geq \bar{P}$ for $t \geq 1$. If the permanent increase had been anticipated then the effects on asset prices and interest rates can be analyzed in a manner identical to that of a transitory change in θ_1.

One other interesting comparative static result that can be obtained from the land model is to suppose that the weather factor affects the marginal product of labor and leaves unchanged the rental rate on land itself.[11] In this case an increase in ω_1 will have qualitatively the same effects on the rate of interest in period 1: Using (3.4), $R_1 = u'(y_1)/\beta u'(\bar{y}) < \bar{R}$. Because $\theta_t = \bar{\theta}$ in all t this implies that $P_1 > \bar{P}$ as well. If ω_1 is anticipated prior to period 1, then the rate of interest in period zero rises in this case as well as when the dividend was increased, and P_0 remains ambiguous depending upon the elasticity of intertemporal

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[9]In the case of CRRA preferences, using (3.13) gives $dP_t/P_t = \sigma(dc_t/c_t - dc_{t+1}/c_{t+1}) + \dfrac{dP_{t+1} + d\theta_{t+1}}{P_{t+1} + \theta_{t+1}}$. Using this relationship, sign (dP_0/P_0) = sign$[1-\sigma(dy/y)(\theta/d\theta)]$. Since $d\theta/\theta > dy/y > 0$, this will be negative for a sufficiently large value of σ.

[10]For all $j < 0$, $y_j = y_{j+1} = \bar{y}$. Therefore $R_j \equiv (P_{j+1} + \theta)/P_j = (\bar{P} + \theta)/\bar{P}$. Assume $j = -1$. Then $P_{-1} = [(P_0 + \theta)(\bar{P} + \theta)]/\bar{P}$. Therefore either $P_0 > P_{-1} > \bar{P}$ or $P_0 < P_{-1} < \bar{P}$. The same argument holds for any $j < -1$.

[11]To model this suppose $y = f(k^{\ell}, \varepsilon)$, $f^{\ell}_{k\varepsilon} = 0$. This is qualitatively the same as supposing Harrod neutral technical change.

substitution. What is important to note about these results is that asset prices and interest rates can fluctuate even in the face of a constant dividend stream.

Suppose next that the increase in the marginal product of labor was permanent. Then $R_t = \bar{R}$ for all $t \neq 0$, and for $t=0$, $R_0 = u'(y_0)/\beta u'(y_1) > \bar{R}$ as in the transitory case. Because $R_t = \bar{R}$ and $\theta_t = \theta$ for all $t \geq 1$ therefore $P_t = \bar{P}$ for all $t \geq 1$. But this means that, if the income change is anticipated, $P_0 = (\bar{P} + \theta)/R_0 < \bar{P}$. Therefore the permanent increase in the marginal product of labor cause asset prices to fall in anticipation of the shock, but does not affect prices after the event.

3.5 Variable Time Preference

One aspect of the previous models is that, with a constant rate of time preference, either the price or quantity of assets must adjust to bring the rate of interest in line with δ (adjusted for population growth). Occasionally we are concerned about adjustment when R is fixed and, for all practical purposes, $\partial R/\partial k = 0$. This is often the case when small open economies are modeled. To be able to discuss equilibrium under these circumstances it is useful to have a model of preferences where the effective rate of time preference is variable. At least two methods have been proposed to provide this description. Uzawa (1968) makes the rate of time preference depend upon future levels of utility, ie, $\delta_t = \delta(\int(u_{t+i})di)$ where $\partial\delta/\partial u > 0$. Thus, as agents accumulate assets and increase consumption, δ rises, decreasing the willingness of agents to save and ultimately bringing the rate of growth of assets in line with the growth of the population. The second approach, used by Kurz(1968) and Fried(1984) supposes that assets enter the instantaneous preference function, eg, $u = u(c_t, a_t)$. This generates an Euler condition of

$$\dot{\lambda}/\lambda = -(r_t - \delta + u_a/u_c).$$

Thus, as assets are accumulated, u_d/u_c is altered changing the "effective" rate of time preference.[12]

3.6 Uncertain Lifetimes

One aspect of the human condition is that the time of our death is uncertain. In the context of the dynastic agent model, one cannot be sure if a family could maintain itself for ever as well. Blanchard (1985) has developed an innovative model that provides the economist the ability to examine the effects of uncertain lifetimes for decision units within the context of certainty models. In it he supposes uncertain life times for individual agents but aggregate certainty for the population as a whole. Throughout their lives agents in this model face a constant instantaneous probability of death of p so that, at any date, t, all agents irrespective of birth date have the same expected future life expectancy. Because, in the aggregate, there is no uncertainty about the total population, life insurance companies can form to take on the agent specific risk of death. The contracts offered are the reverse of what one finds at existing life insurance companies. Specifically the contract offered is one where the company pays the agent p unit of output per instant to have claim on one unit of capital if the agent dies. If an agent survives period t then, for each unit of capital insured, he receives (r_t + p) units of goods and, if he doesn't survive, the insurance company receives all his insured assets. Because an agent has no offspring he wishes to leave a bequest to, he would take insurance on all his assets.[13]

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[12]In both this and the Uzawa case, a stable steady state requires that the effective rate of time preference be increasing in wealth. To see this suppose an equilibrium exists with (r,a) interest rate and asset holdings and suppose $\partial\delta/\partial a < 0$. Then a windfall increase in assets at date t will cause $\delta_t < r$ providing an even greater incentive to save. Thus an equilibrium with $\partial\delta/\partial a < 0$ will be unstable if r is constant.

[13]In the case of borrowing no one would lend to an agent who was not insured, so he would pay p to the insurance company for each unit borrowed in exchange for the insurance company making good on his debts upon his deaths.

Assume the total population is constant. At date t an agent born at date s wishes to maximize:

$$3.16) \qquad {}^{s}\phi_t = E_t \int_t^{\infty} u({}^{s}c_v) e^{\delta(t-v)} dv$$

subject to

$${}^{s}\dot{a}_t = (r_t + p)\,{}^{s}a_t + {}^{s}w_t - {}^{s}c_t$$

where the superscript "s" refers to the birthdate of the agent. To rule out infinite (insured) borrowing; the transversality condition is

$$\lim_{v \to \infty} {}^{s}a_v e^{-\int_t^v [r_\mu + p] d\mu} = 0$$

in which case

$$\int_t^{\infty} {}^{s}c_v e^{-\int_t^v (r_\mu + p) d\mu} dv = {}^{s}a_t + {}^{s}h_t$$

In the log linear case consumption is[14]

$${}^{s}c_t = (p+\delta)({}^{s}a_t + {}^{s}h_t).$$

In turn, aggregate consumption will be a linear function of aggregate wealth, $a_t + h_t$:

$$3.17) \qquad c_t = (p+\delta)(a_t + h_t),$$

where

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[14]The log linear case is used as an illustration. However, to get closed form solutions, it is necessary that preferences be homothetic to permit aggregation in the face of different wealth levels for different generations. The family of CRRA utility functions meets this need so that aggregate consumption at t, c_t, is

$$c_t \equiv \int_{-\infty}^{t} {}^{s}c_t \, p e^{p(s-t)} ds = \kappa(r,p,\sigma)(a_t + h_t)$$

where

$$\kappa(r,p,\sigma) = 1/\int_t^{\infty} e^{\sigma^{-1} \int_t^u [(1-\sigma)(r_\mu + p) - (\delta + p)] d\mu} dv$$

$$h_t = \int_t^{\infty} \omega_v e^{-\int_t^v (r_\mu + p)d\mu} dv$$

when all agents have the same endowment income, and

$$a_t = \int_{-\infty}^{t} {}^s a_t p e^{p(s-t)} ds.$$

Note that non–asset income is discounted at the rate (r_t+p) whereas asset income is discounted at r_t. Alternatively

3.18) $\dot{h}_t = (r_t+p)h_t - \omega_t$ and

3.19) $\dot{a}_t = r_t a + \omega_t - c_t$.

That is, while for the individual ${}^s\dot{a}_t$ depends on $(r_t+p){}^s a_t$, in the aggregate, $\dot{a}_t$ grows at the rate $r_t a_t$ since pa_t is a transfer through the insurance program.

To describe the steady state in the capital model assume $a_t = k_t$ and $\omega_t = w_t$ and have technology given by (2.2)–(2.4). Differentiating (3.17) and substituting in (3.18) and (3.19) then gives

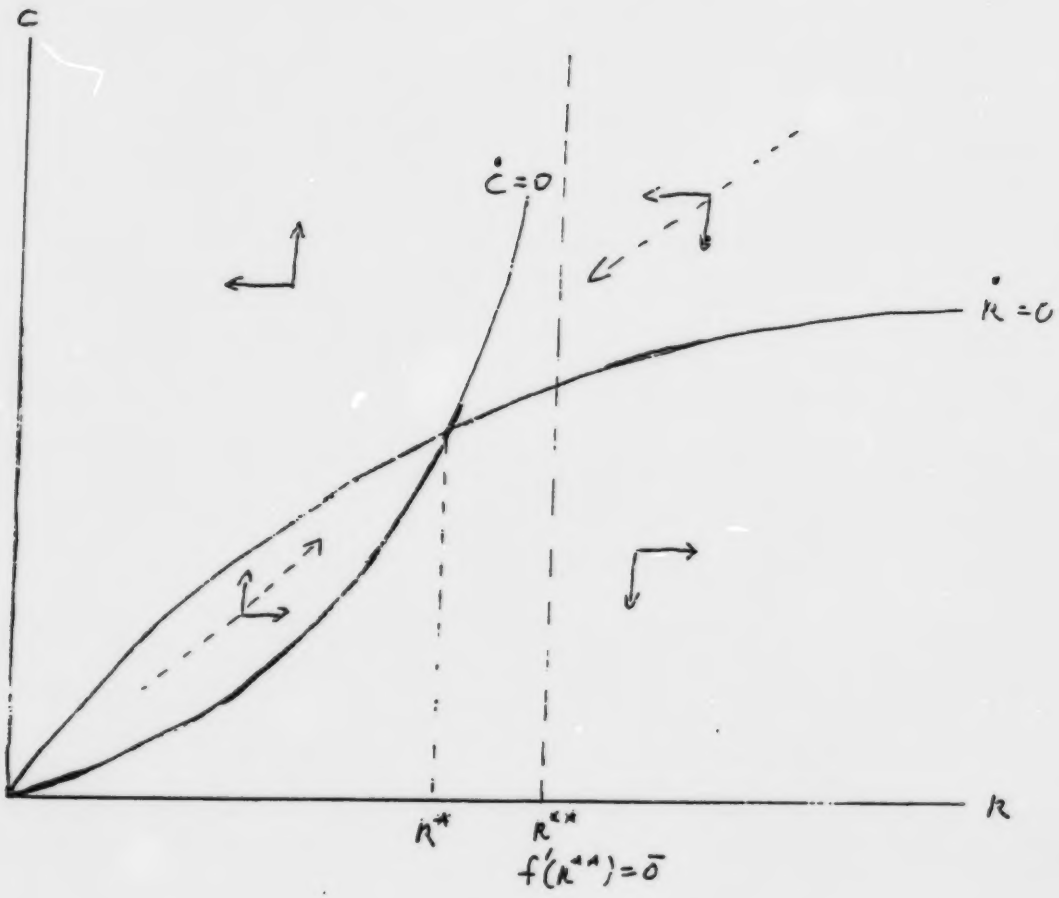
3.20) $\dot{c}_t = (f'(k_t) - \delta)c_t - p(p+\delta)k_t$

the phase diagram for (3.20) and (3.19) is shown in Figure 3.3. A saddle point equilibrium exists with $f'(k^*) = r > \delta$.

The reason why the rate of interest is greater than δ is directly related to assumptions about future endowment income. In the steady state h is constant throughout the agent's life. In order to get positive aggregate capital, $r > \delta$. That is, in a steady state new entrants must want to accumulate assets or else, because the (wealthy) old are dying off, the capital stock would shrink. Furthermore, in the aggregate, this shrinkage occurs at $r < \delta$ rather than $(r+p) < \delta$ since pk is obtained from transfers from the grave.

It is not however necessary that $r > \delta$ in the steady state under all assumptions. Blanchard shows, for instance, that if endowment income for the agent declines through time, then $\dot{c}_t$ becomes

FIGURE 3.3



Equilibrium with uncertain lifetimes

3.21) $\dot{c}_t = (f'(k) + \alpha - \delta)c_t - (p + \alpha)(p + \delta)k_t$

where α is the (exponential) rate of deterioration of individual labor supply. In this case r can be less than δ if α is sufficiently great.

There are at least two principles that can be drawn from Blanchard's analysis. First, all else equal, the longer the agent's expected lifetime the lower will be the steady state rate of interest. In Blanchard's model, $1/p$ is the expected lifetime of an agent and, from (3.20), a decrease in p (increasing the average age) increases k, decreasing the rate of interest, $r = f'(k)$. In effect, if the agent "lived forever" he would accumulate to the point where a further temporary decrease in consumption now would not be worth a permanent increase in consumption forever. With a finite lifetime that same temporary drop would increase future consumption only for as long as he remained alive and would not be valued as highly, so the agent would accumulate less than his longer lived counterpart. The second point to note is that, from (3.21) the more is income concentrated in the present (the higher is α), the lower will be the equilibrium rate of interest. In this formulation this is because the "effective" rate of time preference is δ–α, and an increase in α lowers this rate of time preference. More intuitively, the decrease in human wealth is compensated for by an increase in the demand for physical assets in an attempt to maintain the same consumption stream, in the process lowering the marginal product of capital and the rate of interest.

Blanchard's modification of the basic model of infinite lived representative agents provides additional insight into the determination of interest rates in the steady state. Indeed, the basic model is simply the limiting case as p approaches zero. As such it can address a variety of issues that are difficult to model in the basic case. Nonetheless, one issue that remains unexamined in this framework is heterogeneity of the population and the effect of this on interest rates. One method of modeling this heterogeneity in an intertemporal framework is through the use of overlapping generations models. We next turn to the work that has been done using this type of model.

3.7 The Overlapping Generations Model

3.7.1 Basics

Consider a world where agents have finite lives, and planning horizons, of two periods.[15] That is, in (3.1), $T=2$. A typical agent of generation t works, pays taxes, if any, consumes and accumulates capital in period t when she is young. When she is old, in period $t+1$, she does not work but receives the return on her assets, and sells her capital to the next generation in exchange for consumption goods. Lacking a maternal instinct, she leaves no assets to future generations.

For notational convenience, let c_t be the consumption of a young agent of generation t and x_{t+1} be the consumption of an old person of generation t. It is assumed that from one generation to the next the population grows at the rate, $n \geq 0$. Ignoring the existence of the government, the national income accounts are given by

$$Y_t/N_t \equiv y_t = K_{t+1}/N_t - K_t/N_t + C_t/N_t + X_t/N_t$$
$$= nk_{t+1} + (k_{t+1} - k_t) + c_t + x_t/(1+n)$$

where lower case variables are per worker terms rather than per capita. Output is produced using the same technology as in (2.2)–(2.4). Denote the factor price frontier linking w_t to r_{t-1} as $w_t = \Omega(r_{t-1})$, where

$$\Omega' = -k_t < 0, \text{ and } \Omega'' = -1/f''(k_t) > 0$$

The decision problem for a member of generation t is to maximize:

3.1′) $\quad \phi_t = u(c_t) + \beta u(s_{t+1})$

subject to

3.2′) $\quad x_{t+1} = R_t(\omega_t - c_t).$

Again, let $\omega_t = w_t$.

This gives the same first order (Euler) condition as in (3.4): $R_t\beta = \psi(c_t, x_{t+1})$.

Saving, s_t, is seen to depend upon w_t and r_t,

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[15]The seminal work on this model is Diamond (1965).

$$s_t = s(w_t, r_t)$$

where $s_w > 0$ and s_r can be positive or negative.

For an equilibrium in the capital goods market savings, or the supply of capital, will equal the demand for capital described by (2.3), $r_t = f'(k_{t+1})$. Thus

3.22) $$r_t = f'(s(w_t, r_t)/(1+n))$$

describes that equilibrium. We assume the normal case, where the slope of the demand for capital schedule is less than that of the supply schedule and denote equilibrium in the capital market by the relationship $r_t = \Psi(w_t)$, where

$$\Psi' = f'(k_t)s_w/(1+n-f''(k_t)s_r < 0.$$

The evolution of the economy is shown in Figure 3.4 under the assumption that the equilibrium is stable and unique.[16] The economy operates along the factor price frontier, Ω, and the capital market equilibrium curve indicates the evolution of r_t given w_t.

3.7.2 The Land Model

Now replace k_t with k^ℓ. The goods market clearing condition is

$$c_t + x_t = w_t + \theta_t k^\ell = y_t,$$

and asset market clearing requires

3.23) $$P_t k^\ell = w_t - c_t$$

For a steady state, (c_t, x_t, R_t) are constant at $(\bar{c}, \bar{x}, \bar{R})$, and the Euler condition is the same as in (3.4): $u'(c) = \beta R u'(x)$. In this simple framework we can identify c and x as $\bar{c} = w - Pk^\ell$ and $\bar{x} = (P+\theta)k^\ell$. Thus

$$\beta R = \psi(\bar{c}, \bar{x}) = u'(w - Pk^\ell)/u'((P + \theta)k^\ell)$$

which provides a unique solution for r, but does not suggest a unique relationship between r and δ spanning all technologies. Tastes, technology and the rate of time preference will all

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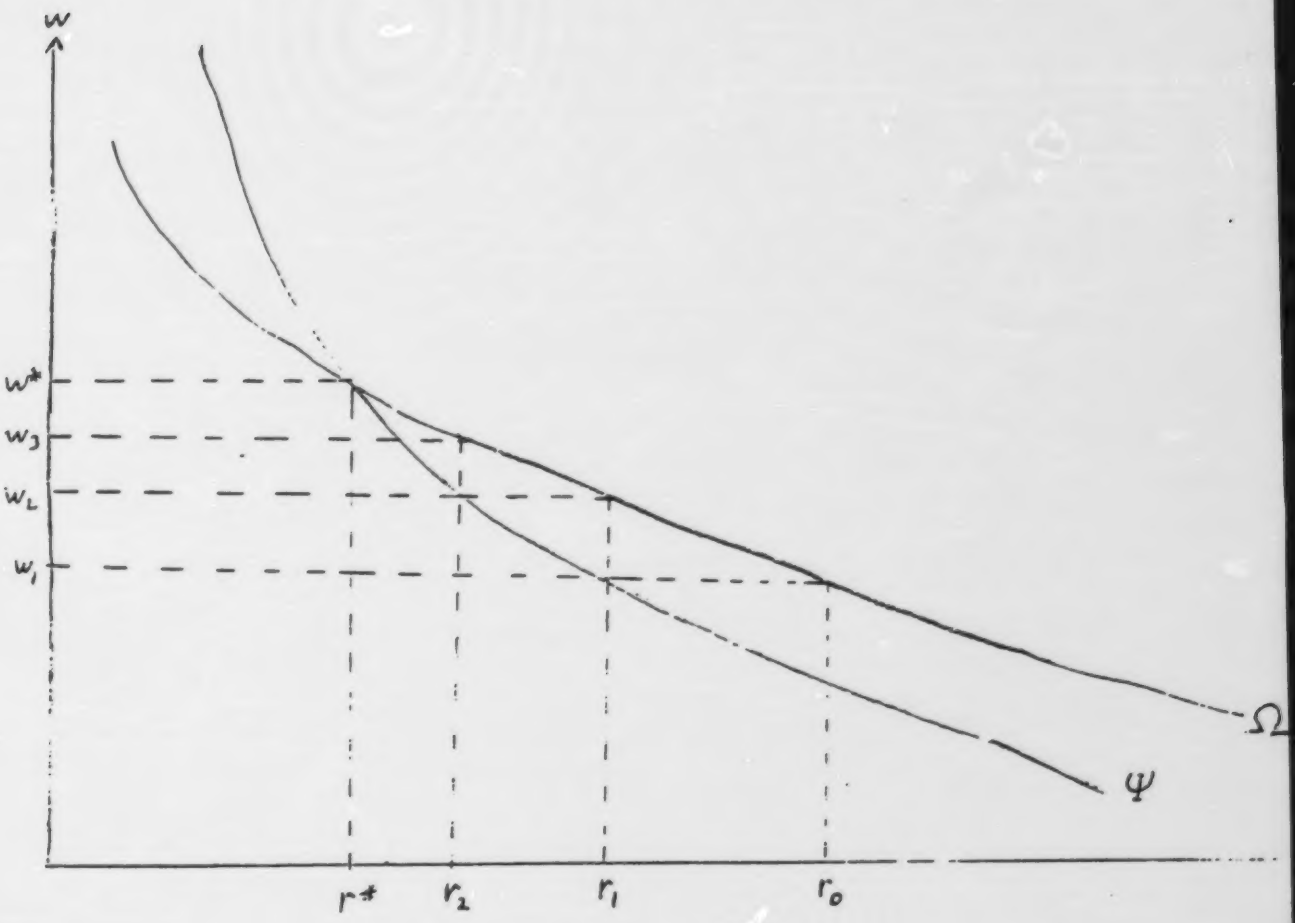
[16]The condition for stability is $0 < dr_t/dr_{t-1} = \Psi'\Omega' = -kf''s_w/(1+n-f''s_r) \leq 1$.

FIGURE 3.4



Equilibrium in the overlapping generations model

have an effect on the equilibrium rate of interest.[17]

In assessing different implications in this and the dynastic model it is important to note that a critical assumption that is often made in OG models but not in the dynastic model is that labor income in old age is zero. Suppose instead that the old and young both worked at the common wage. In this case,

$$c/x = (w - Pk^{\ell})/(w + (P+\theta)k^{\ell}) < 1.$$

Thus, from (3.4), $\beta R = \psi(c,x) > 1$ in the OG framework and, under the assumption of a constant endowment stream, the OG model generates a higher equilibrium R than does the comparable dynastic agent model. Thus, as with Blanchard, finite planning horizons militate against the possibility of dynamically inefficient equilibrium outcomes.

Consider next temporary and permanent shocks to the economy. Suppose first an unanticipated temporary increase in the marginal product of land in $t = 1$. Because the shock is temporary, $P_t = \bar{P}$ for $t > 1$. Further, because the old hold all the land and have a marginal propensity to spend of 1, they receive the entire windfall and consume it all. Thus $P_1 = \bar{P}$ as well, and $R_t = \bar{R}$ for $t \geq 1$. Suppose next that the windfall was anticipated in period 0. Then $R_0 > \bar{R}$ to induce generation 0 to choose a more steeply sloped consumption plan. The effect on asset prices is ambiguous depending on whether $\partial c(R_t, w_t)/\partial R_t \gtrless 0$. The standard case is that $\partial c/\partial R < 0$, in which case $P_0 > \bar{P}$. If the change was anticipated in $t = j < 0$ then, in the standard case, interest rates would rise for the periods $\{j, -1\}$, and $\bar{P} < P_j < P_{j+1} < \ldots < P_0$.[18]

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[17]Consider log linear preferences as an example. In this case $\beta R = x/c = (P + \theta)k^{\ell}/(y - (P + \theta)k^{\ell})$. Solving for P gives $R = (\beta y + k^{\ell}\theta)/w\beta$. If we assume labor's share of output is 2/3, then $R = 1.5 + .5/\beta > 1$.

[18]Consider $j = -1$. Because $c_0 < \bar{c}$, $x_0 = (P_0 + \theta)k^{\ell} > \bar{x} = (\bar{P} + \theta)k^{\ell}$. To induce generation -1 to consume the additional goods in their old age $R_{-1} > \bar{R}$. Therefore $P_{-1} < P_0$. Further, because $\partial c/\partial R < 0$, from (3.21a), $\bar{P} < P_{-1}$. The same argument can be applied for any $j < -1$.

If the temporary shock increased labor income instead of the marginal product of land, a different generation will be affected so the timing of price changes will differ. As before, since the shock is temporary, $P_t = \bar{P}$ for $t > 1$. However $P_1 > \bar{P}$ from (3.23) and the assumption that c and x are normal goods ($1 > \partial c/\partial V > 0$). Thus $R_1 < \bar{R}$. If the shock had been anticipated in $j = 0$, then from (3.4), $R_0 > \bar{R}$ since $P_1 > \bar{P}$ implies $x_1 > \bar{x}$. In the standard case of $\partial c/\partial R < 0$, this also implies that $x_0 > \bar{x}$ implying that, if the system is stable, and the temporary shock is first anticipated in $j < 0$ then $R_1 < \bar{R} < R_j < R_{j+1} < \ldots < R_0$ and $\bar{P} < P_j < P_{j+1} < \ldots < P_1$.

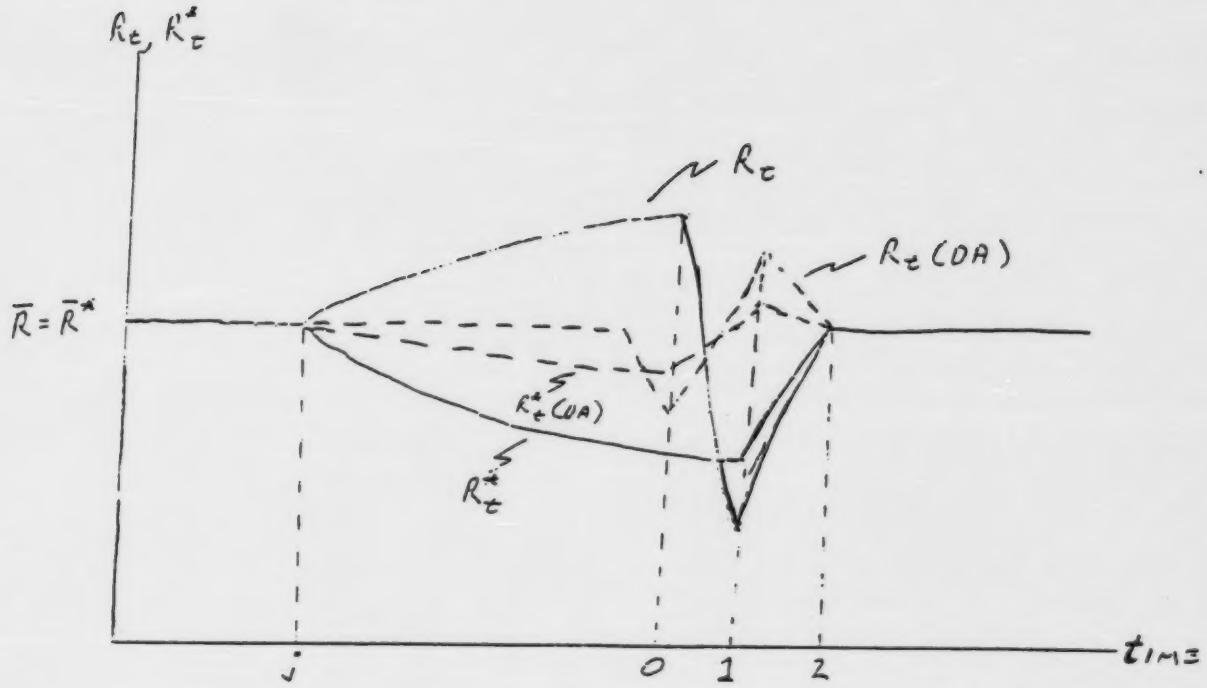
A permanent positive shock to the marginal product of land will increase R_t so that $R_t = R' > \bar{R}$ for all $t \geq 1$. To see this note first that, because all generations born after $t=0$ face the same known economic environment if the equilibrium is stable they will therefore face the same prices. Secondly, suppose, to the contrary, that $R' \leq \bar{R}$. Then $P' > \bar{P}$ and c/x would decrease. But this would violate the Euler condition (3.4). Therefore $R' > \bar{R}$. In the standard case of $\partial c/\partial R < 0$, from (3.23) $P' > \bar{P}$. The effects of an anticipated change would be the same as with a transitory shock. Similarly, for a permanent increase in labor income, $R_t < \bar{r}$ for all $t \geq 1$ and therefore $P_t > \bar{P}$.

Using the case of a temporary increase to labor income there are two important points to note in this analysis. First of all, a given anticipated shock to the economy need not produce a monotonic movement in one period interest rates. In this case interest rates increased up until the shock was experienced, then fell before rising to their steady state level. Second, long term interest rates and short term rates need not move together. The resulting moves of the long term interest rate factor,

$$R_t^* = 1 + \theta/P_t,$$

and in R_t defined by (3.11) are shown in Figure 3.5.

FIGURE 3.5



The effects of a temporary increase in endowment income

Because P_t begins to increase once the shock is anticipated, the long rate falls continuously from $t = j$ through $t=1$. The one period rate rises continuously from $t = j$ until $t=0$, then falls in $t=1$ before returning to its steady state rate. This suggests a cautionary note if the economist wishes to, for instance, determine if the economy is dynamically efficient: In a world of continuous shocks, some anticipated and some not so, focusing on "the" rate of interest will generally be an unreliable guide. The above analysis indicates that it is important to evaluate the entire yield curve or, more generally, all interest rates, in formulating policy actions.

The corresponding behavior of these two rates for the dynastic agent model, denoted $R_t(DA)$ and $R^*_t(DA)$ are shown by the dotted lines in Figure 3.5. The reason for the different behavior in the dynastic agent and OG models is because of the different links between generations in the two paradigms. For the dynastic agent framework consumption is internalized within the "family" so that pressure on consumption is handled through benevolent transfers rather than through market activity affecting prices. No such mechanism exists in the OG model so it matters who receives increased consumption goods and who does not. If an agent is not a recipient of an increased endowment the only way his consumption will change is through the market responses generated by those with changed circumstances. These "distribution effects" occupy center stage in overlapping generations models and many economists, including ourselves, would argue that they play a substantial role in actual economies. The two period lifetime OG paradigm can be viewed as an attempt to capture some of these effects both across generations and within a generation.[19]

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[19]The assumption of two period lifetime is admittedly crude but satisfactory for most of our purposes. Work in public finance and applied general equilibrium have often used more involved OG models where T > 2 and agents within a generation may have different tastes and circumstances. These have been especially useful in analyzing the incidence of alternative tax structures. We shall find the T > 2 models useful in examining incomplete markets and develop a model with T = 3 in Chapter 5. Another possible area where T > 2 period lifetimes may provide insight is in determining the likely effects of the "demographic bulge" on the temporal behavior of interest rates over the next half century.

The dynastic agent paradigm may be quite useful in the analysis of steady state behavior but, because it assumes away these distribution effects, will be incomplete in attempts to fully characterize behavior during a transition or in response to temporary or permanent shocks to the economy.

3.8 Summary

In this chapter we have considered models of individual behavior operating under certainty. This provides a microfoundation for some of the elements of the neoclassical growth theory presented in Chapter 2. We have concentrated on the form of the savings function and primarily around its steady state value. We found that for decision units that can be characterized as having infinite planning horizons, the rate of interest is tied down by the rate of time preference plus the rate of population growth. In this type of model then, for non–negative time preference (a unit of future consumption is never worth less than a unit of current consumption) the issue of dynamic inefficiency never arises.

There are, however, modifications to the model where the steady state rate of interest will diverge from this result. One such case is where assets themselves enter the agent's instantaneous preference function in which case, the "total return" from holding an asset includes not only the measured rate of interest but also the direct marginal utility of asset holding. It is this total return that would be equated with $\delta + n$. In this case it might be possible for the marginal product of capital to be less than n. However, the rationale for capital in the utility function implicitly supposes a market imperfection which will be examined more fully in Chapter 5.

In addition, the work by Blanchard suggests that, if agents have finite expected lifetimes, the steady state rate of interest will be greater than the rate of time preference: all else equal, the shorter the agents lifetime the less willing she would be to accumulate capital. Thus we can argue that, ceterus paribus, the shorter the planning horizon, the greater the steady state interest rate.

The distribution of non–asset income is also important in determining the steady state rate of interest. In particular the earlier in one's lifetime that this endowment income occurs the lower will be the rate of interest. The intuition of this result is simple enough. Individuals are concerned with the entire time path of their consumption. If, when they are old, they receive less income from sources other than capital, they will accumulate more capital to provide them with power to purchase goods at that stage in their life. This result can be confirmed in both the case when lifetimes are known with certainty, as in overlapping generations models, and when lifetimes are uncertain in length, as analyzed by Blanchard.

We have also examined the behavior of interest rates outside the steady state. For the basic model where the decision unit is infinite lived, an unanticipated transitory increase in the marginal product of capital will decrease the rate of interest as agents increase their demand for assets in the period in which the increase in income occurs. This increase in asset demand represents an attempt by agents to smooth their consumption stream by transferring resources into future periods when consumption is expected to be less than in the current period. If the transitory increase had been anticipated, then, the rate of interest would rise in the period immediately prior to the increase in the productivity, again because of the attempt by agents to smooth their consumption stream. If agents, in the aggregate, are unable to transfer consumption goods from one period to the next, perhaps because capital is not reproducible and consumption goods non–storable, then interest rates in earlier periods would not change, although asset prices will move in those periods in the same direction as the change in the period prior to the change in the marginal product. Because asset prices could either increase or decrease, it is not certain whether the capital stock would increase or decrease in response to an anticipated transitory increase in the marginal product of capital. The same qualitative results hold if there is an increase in transitory income from non–capital sources such as wage income. This indicates that there may be no necessary correlation between dividend fluctuations and asset price fluctuations.

An unanticipated permanent increase in the marginal product of capital will not alter the rate of interest but will increase asset prices if capital is not free to adjust; if capital is free to adjust then the rate of interest would rise beginning in the period that the marginal product increased. If the increase is anticipated the price of capital may rise or fall according to whether the substitution effect causing individuals to want to consume more prior to the increased profits to capital is less than or more than the incentive to reap the higher profits of the more productive capital. The greater is the agent's aversion to risk, or the lower his elasticity of intertemporal substitution, the more is the former effect likely to dominate and the price of capital to fall. If it is an anticipated increase in the marginal product of labor, instead of capital, the rate of interest will behave in a similar fashion but, because capital will not become more productive, asset prices will fall in the periods prior to the increased income to the extent that the increase is anticipated.

We also examined non–steady state behavior in the overlapping generations model. Based on the qualitative differences in the results of this model and the basic model we noted a number of important points. First, even for agents with well behaved preferences, anticipated future shocks do not necessarily lead to monotonic movements in one period interest rates. In the case of a temporary increase in labor income, interest rates will rise prior to the increase and then decrease before returning to their steady state values. Secondly, the movement in short and long term interest rates need not move in the same direction in response to anticipated future temporary and permanent changes to capital or labor income. As a consequence, policy made on the basis of the level of "the" interest rate may lead to inappropriate policy choices. In the example considered the one period rate increased but the long term interest rate fell in response to an anticipated increase in labor income. If the capital stock was partially free to respond in this case we would have observed increasing one period rates combined with increased capital accumulation. Finally the overall point of these qualitative differences is that the inclusion or exclusion of distribution effects will matter for

the implications of the type of model the economist uses. If one is led to believe that, as an empirical matter, distribution effects are an important aspect of a certain set of policies then forecasting the consequences of changes in these policies will probably be best done using some variant of the overlapping generations paradigm.

The 1970's were characterized by low real interest rates and the 1980's by high real interest rates relative to the historical levels of the 1960's and earlier. It may be possible to explain some of this movement with changes in anticipations of future income brought on by the oil shocks of 1973–74 and 1979–80 and the apparent collapse of the oil cartel in the early 1980's: The generalized fear in the 1970's that the world would "shortly" run out of oil and people would have a significantly lower standard of living could be interpreted as an expected decrease in future income causing the interest rate to fall; the collapse of the cartel and the low levels of income at the trough of the business cycle in 1982, on the other hand, would lead people to believe that income in the future could only be higher, leading them to attempt to increase current consumption and reducing savings. This could have contributed to the relatively high rates of interest in the 1980's.

The above is certainly not the entire explanation of interest rate behavior over the last two decades. Our analysis also suggests some additional explanations. One such explanation is in the area of distribution effects. There are two possible sources for these effects that may have been more powerful during the 70's and 80's than in previous periods. One source is the "demographic bulge" in North America. Empirically the young and the old have higher average propensities to spend than do cohorts in their middle age, from 45 to 65. As such the high rates in the 80's may be (again, in part) attributable to the absence of these middle aged cohorts relative to the young. This would have the implication that savings levels should rise and rates of interest fall for the remainder of this century. The second distributive source may well have been the government. In particular the rising government deficit has a two fold effect. To the extent that government transfers tend to go to those with low propensities to spend, interest rates would tend to fall. To the extent that the debt servicing falls on this

group, then rates would tend to rise. Thus, rising levels of government debt might tend to generate an interest rate path first of low rates followed by high rates which would in part be reflected in the yield curve. While such a scenario is possible, perhaps the major direction of transfers was toward the elderly in the form of enhanced security benefits. As such, the move to high rates in the 80's may be explainable by this mechanism but not the low rates of the 1970's.

One possible resolution of the puzzle of the low rates of interest in the 1970's is to note that the interest rates that we have been attempting to reconcile are rates of interest on marketable bonds. It is not at all clear that other returns, such as those on equities or on the value of firms themselves, were low in the 1970's. Indeed, Summers (1983) has some evidence that suggests that the low real rates of return in the 1970's was confined to debt and did not extend to less liquid and/or more risky securities. The models we have confined ourselves to in this section cannot address this issue of changes in relative yields. To examine them we must extend our analysis to the issue of risk where interest rate differentials can occur and securities with different risk characteristics are not perfect substitutes.

4 UNCERTAINTY AND INTEREST RATES

4.1 Introduction

In this section we introduce risk into the model. This provides a richer menu of assets with which to concern ourselves. Our interest is to explain the rates of return on broad classes of assets. Specifically our concern is with those classes having a certain payout sometime in the future and those with uncertain payouts. Related to this is one aspect of the literature denoted the "equity premium puzzle": empirically the risk premium on equities is too high to be consistent with other work on consumption behavior.[1] Part of the purpose of this section is to propose possible explanations for this empirical (or theoretical) anomaly.

Having derived formulae for a risk free security and for all wealth, we find it useful to briefly extend the analysis in two additional directions. The first of these is the term structure. We show that the serial correlation of income is important in characterizing the risk premium on risk free securities of differing maturities. We also examine equilibrium returns on uncertain income streams—the portfolio choice problem within the new–classical framework.

4.2 Basics

Work on decision making under uncertainty takes as its starting point that agents wish to maximize their expected utility. Combined with complete contingent claims markets, this hypothesis has been quite powerful in providing financial economists with explicit pricing formulae for a large array of assets. One might tend to dismiss the implications of such work based on the highly unrealistic assumptions of the analysis were it not for the fact that a number of participants in on–going financial markets use these derived relationships on a minute to minute basis to make their purchase and sales decisions.[2] In this section we survey

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[1] See Mehra and Prescott (1985).

[2] We do not have evidence that such agents make above average profits. Similarly we do not have evidence that market participants who use astrology as their market guide do better or worse than average. Economic Darwinism appears to be a rather lax task master.

a few of the fundamental asset pricing relationships that bear on the role of government debt and the cost of capital.

The decision problem the individual agent faces is to maximize the expected value of (3.1):

$$E_0[\sum_{t=0}^{T}\beta^t u(\tilde{c}_t)],$$

subject to the constraint that

4.1) $\tilde{a}_{t+1} = \tilde{R}_t(a_t + \tilde{\omega}_t - \tilde{c}_t),\ 0 \le t \le T,$

where E_t is the expectations operator conditioned on all information available at t, and a $\tilde{}$ over a variable indicates that it is stochastic. At date t, the available information consists of all past history, the current asset holdings and endowments, and the distribution of the underlying shocks that impact on $\tilde{R}_{t+i}$ and $\tilde{\omega}_{t+i}$, all $i > 0$. The Euler equation for this problem is

4.2) $u'(c_t) = \beta E_t(\tilde{R}_t u'(\tilde{c}_{t+1}))$.

As in the certainty case, the RHS of (4.2) indicates the loss from consuming one less unit of consumption now, and the LHS measures the expected gain from increasing asset holdings by that one unit. At the optimum, the expected gain should just equal the expected cost. Equation (4.2) is the basis for analyzing interest rates. However, we find it convenient to express interest rates in terms of the moments of the joint distribution of interest rates and marginal utilities. Accordingly, noting that $E_t(\tilde{x}\tilde{y}) = \hat{x}\hat{y} + \mathrm{cov}_t(\tilde{x},\tilde{y})$, (4.2) can be rewritten as

4.3) $\hat{R}_t = [u'(c_t)/\beta E_t u'(c_{t+1})]\ [1 - \mathrm{cov}_t(\tilde{R}_t, \beta\psi(\tilde{c}_{t+1}, c_t)]$

where $\hat{x} = E_t(x)$ and $\mathrm{cov}_t(x,y)$ is the covariance of x and y conditional on all information available at t.

To analyze the behavior of interest rates we use the simple dynastic agent land model with no endowment income. This corresponds to the "tree" model of Lucas (1978). Suppose that each representative agent begins, at $t=0$, with one unit of land that pays a dividend, $\tilde{\theta}_t$, in period t, $t > 0$. The land can be sold but, because all agents are identical, equilibrium prices will adjust so that each agent is just satisfied holding his one unit of land. Thus $\tilde{c}_t = \tilde{\theta}_t = \tilde{y}_t$, $\forall t$

and (4.3) specializes to :

$$4.4) \quad \hat{R}_t \equiv \frac{\hat{P}_{t+1} + \hat{y}_{t+1}}{P_t} = [u'(y_t)/E_t u'(\tilde{y}_{t+1})\beta][1 - \mathrm{cov}_t(R_t, \beta\psi(\tilde{y}_{t+1}, y_t))]$$

For expositional purposes think of the output per unit of land being

$$y_t = \hat{y}(1 + e\varepsilon_t),$$

where ε_t is a random variable having $E_t(\varepsilon_t) = 0$, e is a scale variable that provides an analog to the degree of volatility of y_t, and $\hat{y}$ is the expected value of y_t. The support for ε_t is such that $1 + e\varepsilon_t$ is positive.

The type of questions we would like to ask concern asset prices and returns if there is a change in either the expected dividend from land or in the riskiness of that return. While there is only one source of uncertainty, (4.4) can be used to describe these effects on a variety of different instruments by modifying the assumptions slightly. For instance, there could be N types of land that provide returns that are stochastically independent or, from ownership of a unit of land, the owner could offer a risk free security and become a residual claimant on the output of the land, generating a risk free security and a (levered) risky asset. We consider riskless assets first.

4.3 The Risk Free Rate of Interest

A risk free asset is a promise to pay with certainty one unit of output one period hence. Its gross return is defined as

$$R_t^f = 1/P_t^f$$

and the net return is $(1-P_t^f)/P_t^f = r_t^f$, where P_t^f is its market price at date t. Because its payment is certain, the conditional covariance, $\mathrm{cov}_t(R_t^f, \psi(\tilde{y}_{t+1}, y_t)) = 0$ and, from (4.4), its return is given by

$$4.5) \quad R_t^f = u'(y_t)/E_t u'(\tilde{y}_{t+1})\beta.$$

To begin, compare R_t^f to the one period rate, R_t^*, in the certain world when $T = \infty$. Suppose the same income in period t and that $\hat{y}_{t+1}$ with uncertainty equals y_{t+1} in the certain

world. Then $R_t^f = u'(y_t)/\beta E_t u'(y_{t+1}) \lesseqgtr R_t^* = u'(y_t)/\beta u'(\hat{y}_{t+1})$ as $E_t u'(y_{t+1}) \gtreqless u'(\hat{y}_{t+1})$. Whether $E_t u'(y_{t+1})$ is greater or less than $u'(\hat{y}_{t+1})$ in turn depends upon the third derivative of the preference function. While economists have no special insight into the nature of preference functions, it appears that the assumption that u''' is positive is less restrictive than many of the assumptions that we have already made about preferences:[3] Indeed, for the entire class of CRRA preferences, u''' is positive. We therefore shall assume that the third derivative of the preference function is positive. Then $E_t u(y_{t+1}) > u'(\hat{y}_{t+1})$ and therefore

$$R_t^f < R_t^*.$$

Figure 4.1 characterizes this situation. Suppose $\hat{y}_{t+1} = \underline{y}$ and there are 2 equally likely states in period $t+1$, (y^-, y^+). Then $E_t(u'(y_{t+1}))$ is equal to the point on the chord connecting y^- and y^+ centered at y. Given the concave marginal utility function $E(u'(y))$ must necessarily be greater than $u'(y)$[4]. Furthermore, under these same assumptions, an increase in the mean preserving spread will decrease R^f. This is seen in Figure 4.1, where changing the possible states to (y^{--}, y^{++}) from $(y^- y^+)$ increases $E_t u'(y_{t+1})$ and leaves $u'(\underline{y})$ unchanged.

The second experiment is to examine the effect of an increase in expected future income. In this case, because the entire distribution of y_{t+1} is displaced upward, the marginal utility of y_{t+1} will be lower for all ε_{t+1}. Thus $E_t(u'(y_{t+1})$ will fall, increasing R_t^f.

4.4 The Term Structure of Interest Rates

The term structure of interest is concerned with the rate of interest, or yield, on risk free securities of different maturities. The starting point in any study of these relative yields is the recognition that the return on an M period bond bears some relationship to the current one

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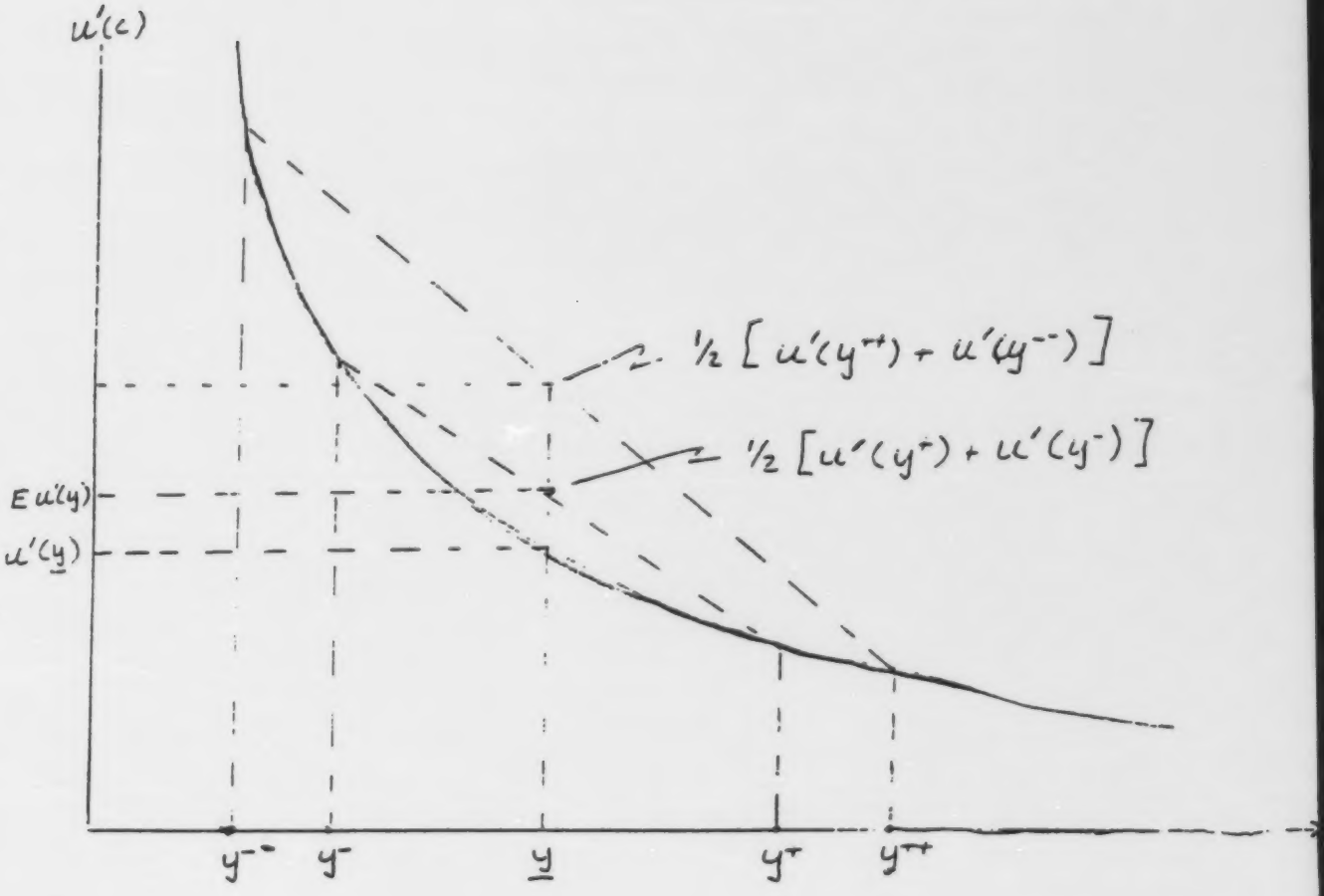
[3]Zeldes (1989a) points out that assuming $u''' > 0$ is no less restrictive than the commonly made assumption of decreasing *absolute* risk aversion.

[4]Abel et al. (1989) provide a case in point when they use the log linear preference function ($\sigma = 1$).

FIGURE 4.1



The effects of increased uncertainty

period rate times the currently expected one period rates extending over the lifetime of the M period bond. The difference between the yield on the M period bond and the (geometric) average of the expected returns on the sequence of one period bonds spanning the same period is called the term premium, T^M. That is,

$$\log T^M = (1/(M+1)) \log R_{t,M} - (\log R_t + \sum_{1}^{M} \log E_t R_{t+i}).$$

where $R_{t,j}$ is the certain gross rate of return on a $j+1$ period bond at date t. To value this term premium consider first a (discount) bond that pays, with certainty, one unit of consumption goods two period from the present. The relevant first order condition for this bond would be:

4.6) $$\begin{aligned} u'(c_t) &= \beta^2 E_t(R_{t,1} u'(c_{t+2})) \\ &= \beta^2 R_{t,1} E_t u'(c_{t+2}) + \beta^2 \mathrm{cov}_t(R_{t,1}, u'(c_{t+2})) \\ &= \beta^2 R_{t,1} E_t u'(c_{t+2}) \end{aligned}$$

where $\mathrm{cov}_t(R_{t,1}, u'(c_{t+2}))$ equals zero because $R_{t,1}$ is known with certainty. Rearranging (4.6) to

$$R_{t,1} = u'(c_t)/\beta^2 E_t(u'(c_{t+2}))$$

and multiplying the numerator and denominator by $E_t u'(c_{t+1})$ gives:

4.7) $$\begin{aligned} R_{t,1} &= [u'(c_t)/\beta E_t u'(c_{t+1})][E_t u'(c_{t+1})/\beta E_t u'(c_{t+2})] \\ &= R^f_t[E_t u'(c_{t+1})/\beta E_t u'(c_{t+2})]. \end{aligned}$$

From (4.5)

$$u'(c_{t+1}) = \beta[E_{t+1} u'(c_{t+2})]R^f_{t+1}.$$

Therefore

$$E_t u'(c_{t+1}) = \beta E_t\{[E_{t+1} u'(c_{t+2})]R^f_{t+1}\}.$$

Using (4.2) and (4.3) this becomes

$$E_t[u'(c_{t+1})] = \beta[\hat{R}^f_{t+1} E_t u'(c_{t+2}) + \mathrm{cov}_t(R^f_{t+1}, u'(c_{t+2}))].$$

Rearranging and substituting into (4.7) then gives:

4.8) $$R_{t,1} = R^f_t[\hat{R}^f_{t+1} + z_{t,1}],$$

where

$$z_{t,i} = \mathrm{cov}_t(R^f_{t+i}, u'(c_{t+i+1}))/\beta E_t u'(c_{t+i+1})$$

is the forward premium on a riskless bond maturing $i+1$ periods from the current date, t. More generally, the price of a discount bond paying one unit of output with certainty $M+1$ periods from the current date is

$$P^f_{t,M} = 1/R_{t,M}$$

where

$$R_{t,M} = R^f_t \prod_{i=1}^{M} (\hat{R}^f_{t+i} + z_{t,i}).$$

The forward premium is, in principle, measurable once a forecast of future one period rates is made, with $z_{t,i} = (P^f_{t,i-1}/P^f_{t,i}) - E_t R^f_{t+i}$, and where $P^f_{t,i-1}$ and $P^f_{t,1}$ are the period t prices of two adjacent risk free discount bonds. The term premium is an average of these forward premiums. Finally, since $R^f_{t+i} = u'(y_{t+i})/E_{t+i}u'(y_{t+i+1})$, $z_{t,i}$ can be rewritten as:

$$z_{t,i} = \mathrm{cov}_t(u'(y_{t+i}), u'(y_{t+i+1}))/(\beta^2 E_t u'(y_{t+i+1})).$$

This clearly brings out the dependence of the forward premium upon the serial correlation of income about its trend. If income is positively serially correlated, the premium will be positive.

4.5 Risky Dividend Streams

Next consider the case where there is a single risky asset having a gross stochastic return $\tilde{R}_t$. From (4.4), this return can be decomposed into two parts, the risk free rate, R^f_t and a risk premium, $\hat{z}_t$, where

4.9) $$\hat{z}_t \equiv \hat{R}_t - R^f_t = -R^f_t \mathrm{cov}_t(\tilde{R}_t, \psi(\tilde{y}_{t+1}, y_t)).$$

This premium is necessarily positive if the security's dividend is the only source of uncertainty

in the economy. An increase in the mean preserving spread will generally increase the risk premium,[5] but does not necessarily increase the required return on the risky asset. This is because the decrease in the return on the risk free security may more than compensate for the increase in $\hat{z}_t$. Work by Barsky (1986), Abel (1988) and others indicate that, in the CRRA case, if the coefficient of relative risk aversion, γ, is greater than one, the required expected return will fall, increasing the asset price. The reason for this is that, if agents are quite risk averse, the increased uncertainty induces them to increase precautionary saving in an attempt to avoid the increased possibility of an undesirable state of nature in the future. This increased attempt to save drives up the price of the risky asset.

An alternative explanation compares this result to the certainty case. In a certain world a high γ ($\sigma < 1$) meant that an increase in R_t decreases savings. With uncertainty, the increase in risk can be viewed as a decrease in the certainty equivalent rate of return, as indeed occurs to R_t^f. The fall in the certainty equivalent rate also will cause "consumption smoothers" ($\sigma < 1$) to save more. This increase in the supply of loanable funds relative to demand depresses the entire interest rate structure. On the other hand, agents with high substitution preferences decrease saving because the return becomes less attractive with the increased risk.

This ambiguity carries over to the (stochastic) steady state rate of interest relative to the rate of time preference. Suppose we were in a steady state under certainty with $r = \delta$ and conduct the experiment of introducing a small amount of uncertainty into the dividend stream from land. If $\sigma < 1$, then the demand for assets would rise, indicating the steady state rate of interest would decrease. The converse would occur if $\sigma > 1$. With log linear preferences the rate of interest would remain unchanged at δ.

A decrease in the expected future stream of dividends works in much the same way as an increase in dividend volatility. The decrease in future expected earnings cause the more

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[5]Abel (1988) provides a counter-example.

risk averse ($\gamma > 1$) agents to compensate for that fall in future income by increased saving that more than compensates for the fall in the dividend stream. The case of no change in the asset price occurs with log linear ($\gamma = 1$) preferences, while agents having low levels of risk aversion would, in equilibrium, be unwilling to pay a higher price for a lower expected dividend stream.

Suppose next that there are two sources of income instead of one. How are they to be priced? Rewriting (4.1) in terms of asset prices instead of returns, the constraint the agent faces is

$$P_{1t}s_{1t+1} + P_{2t}s_{2t+1} = \sum_{i=1}^{2}(P_{it} + \theta_{it})s_{it} - c_t$$

where s_{it} is the number of shares of the i^{th} security held at the start of period t, and the Euler condition for the i^{th} security is

$$\begin{aligned} P_{it} &= \beta E_t[(\psi(c_{t+1},c_t)(\tilde{P}_{it+1} + \tilde{\theta}_{it+1})] \\ &= \beta[(E_t u(c_{t+1})/u'(c_t)\cdot(\hat{\tilde{P}}_{it+1} + \hat{\tilde{\theta}}_{it+1})] + \mathrm{cov}_t(\psi(c_{t+1},c_t), (\tilde{P}_{it+1}+\tilde{\theta}_{it+1})) \end{aligned}$$

or

$$\hat{\tilde{R}}_{it} = [u'(c_t)/E_t u'(c_{t+1})\beta][1 - \mathrm{cov}_t(\psi(\tilde{c}_{t+1},c_t), \tilde{R}_{it})], \qquad i=1,2.$$

Note that in this model, $\tilde{c}_{t+j} = \tilde{y}_{t+j} \equiv \tilde{\theta}_{1t+j} + \tilde{\theta}_{2t+j}$ $\forall j$.[6] Thus the rate of return on any risky asset is the risk free rate, $u'(c_t)/E_t u'(c_{t+1})\beta$, plus the risk premium, $\hat{z}_{it}$, that depends upon the covariation of consumption and the own rate of return.[7]

Suppose that θ_{1t} and θ_{2t} are independent of one another and let there be an increase in the mean preserving spread of θ_1. As before, in the CRRA case this will decrease the risk free rate of interest. It will also generally increase the risk premium on asset 1 and, depending on the elasticity of intertemporal substitution, may lead to a rise or fall in the price of the asset. Barsky has also shown that the rate of return on the second asset will fall irrespective of value

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[6]It should also be pointed out that one of these assets can be interpreted as human capital generating a dividend in period $t+i$ of $\tilde{\omega}_{t+i}$, all $i \geq 0$.

[7]Breeden (1979) has generalized this analysis to many assets, consumption goods and investment opportunities in a continuous time framework.

of relative risk aversion. If you will, given the independence assumption, the increase in precautionary savings is directed toward both the risk free asset and the now *relatively* less risky assets, causing their expected returns to decline. The same qualitative result also occurs from a decrease in the expected future dividends on asset 1.

One can place arbitrage pricing theory in this context. According to this hypothesis, any risky security can be decomposed into a portfolio of M different primary factors that are stochastically independent. Given m ($> M$) securities comprised of these primary factors, arbitrageurs will insure that each security is appropriately priced in terms of these M factors, for otherwise excess profits would accrue to the arbitrageur at no cost nor risk. Incorporating this into the above analysis would have agents demand primary factors which are embedded in existing securities. A change in the expected return, or in the mean preserving spread, of one of these factors would then impact on actual securities prices via the pricing of all factors and the no arbitrage condition.

An apparent interest rate "puzzle" in the actual data for the U.S. is that the equity premium, $\hat{z}$, appears to be "too large".[8] Using an artificial economy framework, Mehra and Prescott calculate that, to explain the actual risk premium, the coefficient of relative risk aversion would have to be far in excess of 10, which is the upper limit obtained from information on consumption behavior. One possible explanation for this is that the risk premium is based upon actual returns on equities and these equities represent only a portion of the value of the firm. By the Modigiliani Miller theorem, the value of the firm, v_t^*, which is the sum of the value of its equity and its debt, $v_{et} + v_{bt}$, is independent of how the firm is financed. Let R_t^e and R_t^* denote the returns on actual equity and the total value of the firm. Then,

4.10) $$R_t^f v_{bt} + R_t^e v_{et} = R_t^* v_t^*,$$

and define η_t as the debt equity ratio v_{bt}/v_{et}. Using (4.9) and invoking the Modigliani Miller

[8]See Mehra and Prescott (1985).

Theorem, (4.10) can be rewritten as

$$R_t^f\eta + (R_t^f + \hat{z}_t^e) = (R_t^f + \hat{z}_t^*)(1+\eta)$$

or

4.11) $$\hat{z}_t^* = \hat{z}_t^e/(1+\eta).$$

A debt equity ratio of one would mean that use of actual market data on equity would overstate the risk premium for the underlying firms by 50%. For the consumption based model, however, the appropriate leverage measure relates to risky assets relative to all assets, and the structure of the firm may be such that it absorbs some of the risk that would have been borne by labor and other factors of production. Suppose, as an extreme, that the only risky asset, tangible or intangible, is corporate profits. In the U.S. corporate profits account for an average of approximately 10% of output. Under this assumption the debt–equity ratio is 9. Thus the actual risk premium overstates the U.S. economy's risk premium ratio tenfold. To get a feeling for the magnitudes involved, Mehra and Prescott calculate the average real return on the S & P 500 composite index over the period, 1889–1979, at 6.98%, and the real return on a short term bill at .8%. This means a measured equity risk premium ($\hat{z}_t^e$) of 6.18%. They also argue that the maximum feasible value their model admits is $\hat{z}^* = .35\%$. Using the debt equity ratio of 9 and all risk concentrated in equities, this increases substantially the implied $\hat{z}_{et}$ to 3.5%, but it clearly remains far below the actual measure of the risk premium.

There have been other attempts to explain this large risk premium within the framework of complete markets and expected utility maximization models. Rietz (1988), using a variant on the "Peso problem", posits the existence of a low probability state of nature where there is a 1% chance of 25% fall in consumption. However, for the U.S., even in the Great Depression consumption never fell by more than 9%. Thus, some in the profession maintain some cynicism toward this resolution. Macklem (1989) has separated consumption into durable and nondurable expenditures in an attempt to increase the feasible risk premium. While the premium increased with this formulation, the feasible set of premiums remained far below the actual premium even adjusting for a debt–equity ratio of 9.

4.6 Non Von Neumann–Morgenstern Preferences

While no doubt there will be further attempts to explain the very large gap between equities and risk free securities using expected utility maximization in an Arrow–Debreu complete markets paradigm, we would suspect that the consensus view on the subject is that a more fruitful line of inquiry is to abandon some of the assumptions of that paradigm in attempting to explain the equity premium. One such attempt has been to jettison the link between portfolio choice and consumption behavior by changing the assumption that agents wish to maximize expected utility. We follow Epstein (1988) and Epstein and Zin (1987a,b) in examining this alternative.

In this framework the agent computes a certainty equivalent of random future utility, μ_t, and this combined with current consumption—which is not random—forms an aggregator with which to compute utility. As a specific example, let

$$\mu_t = \begin{cases} [E_t U_{t+1}^{\alpha}]^{1/\alpha} \text{ if } 0 \neq \alpha < 1 \\ \exp[E_t \ln U_{t+1}] \text{ if } \alpha = 0, \end{cases}$$

and the aggregator function, $W(c_t,\mu_t) = U_t$ take on the CRRA form,

$$W(c,z) = [c^{(1-\gamma)} + \beta z^{(1-\gamma)}]^{1/(1-\gamma)} \qquad c,z \geq 0 \quad \gamma > 0 \quad 0 < \beta < 1,$$

so that utility is defined recursively by means of

4.12) $$u_t = [c_t^{(1-\gamma)} + \beta(E_t u_{t+1}^{\alpha})^{(1-\gamma)/\alpha}]^{1/(1-\gamma)}.$$

In the deterministic case this gives

$$U_0(c_0,c_1,\ldots) = \sum_{0}^{\infty} (\beta^t c_t^{(1-\gamma)})^{1/(1-\gamma)}$$

which is an intertemporal CRRA utility function with elasticity of substitution, $\sigma = 1/\gamma$. In the case where $\alpha = (1-\gamma)$, (4.12) becomes

$$U_0(\ ) = (E_0[\Sigma \beta^t \tilde{c}_t^{(1-\gamma)}]^{1/(1-\gamma)}$$

which is a common transformation of CRRA preferences. The coefficients γ and α separate the substitution and risk aspects of this set of preference functions. In particular, γ is an index of intertemporal substitutability of consumption, the smaller is γ the more the agent will alter his consumption in response to changes in the rate of interest. α, on the other hand, is an index of risk aversion: the greater is α the less risk averse the agent. This can be seen in the construction of the certainty equivalent of future expected utility: the greater is α, the greater the certainty equivalent value $E_t(U^{\alpha}_{t+1})$. For the one asset case this formulation of the decision problem gives the Euler condition

$$\beta[E_t(c_{t+1}/c_t)^{\gamma/(1-\gamma)}R_t^{\alpha/(1-\gamma)}] = 1.$$

In the M asset case:

$$\beta[E_t[(c_{t+1}/c_t)^{\alpha(\gamma)/(1-\gamma)}R_t^{m(\alpha-(1-\gamma))/(1-\gamma)}R_t^j)]^{(1-\gamma)/\alpha} = 1, \quad j=1,...M,$$

where

$$R_t^m = \sum_i^M \zeta_{it}^* R_t^i$$

is the gross return on the optimal market portfolio, $(\zeta_{it}^*,...,\zeta_{Mt}^*)$. This compares to

$$\beta E_t[(c_{t+1}/c_t)^{\gamma}r_t^j] = 1 \qquad j=1,...M$$

when $\alpha = (1-\gamma)$.

Some empirical work by Epstein and Zin (1987b) suggests that an elasticity of substitution in the range .1 to .5, and a coefficient of relative risk aversion of around 1.0 provides a reasonably good fit to the consumption and asset data.[9]

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[9]In (4.12) the elasticity of substitution, $\sigma = \frac{1}{\gamma}$ and the coefficient of relative risk aversion corresponds to $1-\alpha$.

4.7 Saving and Dynamic Inefficiency

So far our analysis has supposed that the stock of capital is fixed in supply. This means that net saving for the economy is zero. We argued in section 3 that, given the negatively sloped asset demand function, changes in asset prices with no change in capital can be interpreted as a change in the quantity of capital in the same direction at a given asset price. Thus, in our analysis in this section, increases in asset prices will correspond to increases in saving and positive capital accumulation if capital is reproducible. It is, however, important to recognize that the asset price that is relevant for the investment decision is the price of the firm rather than that of risk free debt. This follows directly from the Modigliani–Miller theorem. For suppose the new investment was financed at the risk free rate and provided an expected return equal to this rate. Then the risk of the investment would fall on equity holders with no compensating increase in expected return. Therefore, the value of equity would fall and the increase in the value of the firm would be less than the price of the new capital purchased. Thus the increase in capital reduced the wealth of the original stock and bond holders violating the value maximization objective of the firm (as well as putting some of the firms managers out on the street).

Similarly, dynamic inefficiency occurs if the cost of capital to the firm is less than the population growth rate and not if the risk free rate is less than n. To see this suppose the economy is in a (stochastic) steady state with $R^* > 1 + n > R^f$, where R^* is the cost of capital to the firm and equal to the expected marginal product of capital. Then reducing the capital stock by one unit per worker forever would decrease the expected level of output per worker by $R^* - 1$ and decrease expected consumption by $R^* - (1+n) > 0$ per period. Thus, some agents must necessarily have lower consumption and the new equilibrium could not have caused a Pareto improvement. A second way to see this is to suppose that the number of equity shares is reduced, and bond shares initially increased, leaving the value of the firm unchanged. Then ask whether households would want to reduce the level of the capital stock. The answer to that question is that they would not and that the value of the firm would be unchanged. To see

this, note that if the decrease in equity was spread over all equity holders and they received the value of shares lost with an equal value of bonds their choice set would be unchanged in all states of nature and they would be indifferent between the situation with more or less debt. This argues that agents willingly bear the risk of the total return to the firm and would lose expected utility if the stock of capital was somehow reduced.[10]

4.8 Summary and Implications

Risk provides a richer environment in which to evaluate interest rates if only because different sources and levels of risk on different securities will generally imply imperfect substitutability of assets. We have derived the equilibrium expected returns on different securities in two dimensions, term to maturity and uncertainty regarding the stream of dividends and payouts. For the term structure, we find that forward premiums depend upon the covariance of consumption about its trend: If it is positively correlated, these premiums will be positive and long term securities will sell at prices below the expected price of a sequence of one period bills. For securities with uncertain payouts the risk premium depends positively on the covariation of assets's rate of return with the marginal utility of consumption.

Changes in perceived risk as described by an increase in the mean preserving spread of consumption, or changes in the expected stream of future dividends will alter relative yields. In particular an increase in the mean preserving spread in the payout, or decreases in expected future dividends, on a given asset will decrease the expected return on the risk free security and risky securities that are not composed of the risk factor that has increased. The price of the risky asset itself will increase or decrease depending on whether the elasticity of

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[10]This is most easily seen when $n = 0$ and $R^f < 1$. In this case the additional debt held requires debt holders to pay interest to equity owners. Then the increase in debt of the firm simply means that equity holders pay themselves more and no real changes take place. Furthermore, so long as $\hat{R}^{A*} > 1$, there does not appear to be any reason why the owners of the firm could somehow be made better off by being forced to own a smaller capital stock.

intertemporal substitution is less than or greater than one. This latter result is consistent with our analysis in the certainty case.

The addition of risk to the model provides a more coherent interpretation of the behavior of (relatively) risk free rates over the last two decades than does the certainty model developed in section 3. If it is indeed the risk free real rate that was low in the 70's and high in the 80's then the ambiguity surrounding anticipated permanent and transitory shocks that was noted in the certainty case is resolved. An anticipated decrease in the expected marginal product of either labor or capital unambiguously decreases the risk free rate of interest irrespective of what might occur to the overall rate of return on risky assets in the economy. Thus the uncertainty associated with the oil shock in 1973 would reduce risk free rates in the 1970's, and the breakdown of the cartel in the early 80's would increase them in the 1980's. The increased confidence about further increases in output as the world economy came out of the largest economic contraction since the 1930's no doubt also contributed to the higher real rates in the 1980's. Furthermore, to the extent that the perceived riskiness of future income levels increased in the 70's and decreased in the 80's, the implied time path of interest rates can be further explained. While such psychological perceptions of risk may not have been realized ex post, these "speculative bubbles" appear to have exerted a very real affect on the risk free real rate of interest.

Finally, the introduction of risk does not qualitatively alter the condition defining dynamic inefficiency: If the rate of output growth is greater than the cost of capital to the economy, the stochastic equilibrium is dynamically inefficient. If the expected marginal product of capital is greater than the expected rate of output growth, which in turn is greater than the risk free real rate of interest the economy is dynamically efficient: A reduction in the capital stock will decrease the expected utility of some agents without providing the resources to compensate them.

While the introduction of risk into our analysis would appear to enhance the ability of consumption based models to explain the behavior of interest rates, the analysis does not yet appear to be complete. Work using artificial economies note the inability of these types of models to explain more than half of the risk premium of equities even correcting for the leverage implicit in the developed economies. We would argue that the essential ingredient that is missing in these models is the effect of incomplete, or costly to operate, capital markets. These complications will be considered next.

5 INCOMPLETE MARKETS

5.1 Introduction

In this section we survey the effects of incomplete markets on interest rates. Our concern is particularly with the problem raised in the previous section, that consumption based models of asset pricing have performed poorly in explaining the relative rates of returns on riskless and risky securities—the equity premium. In this section we examine the effects of these incomplete, or costly to operate, markets on interest rates, first in a certain world and then with uncertainty. In the certainty case we argue that the liquidity services provided by money and other liquid assets, command a premium price over other assets that are more costly to exchange. In the case of uncertainty we examine the effects of the absence of secondary markets on the prices of tradeable assets using the term structure of interest rates as an example. We conclude that the imposition of constraints on asset trades in one market reduces the flexibility of the agent, increasing his demand for flexibility available from other assets. This leads to a decrease in the return on flexible assets and an ambiguous effect on their price volatility. We introduce a futures market/insurance market to explicate the reasons for this ambiguity. We also note that incomplete markets imply that agents are unable to diversify completely and that perceived individual risk will be greater than measured aggregate risk in the economy. Taking account of this inability to diversify may also help explain the magnitude of the equity premium.

5.2 Certainty and Costly Marketing Activity

One issue that has been at the forefront of monetary theory over the last decade has been the question of positive valued money in the face of rate of return dominance by other assets. The explanations for this phenomenon have taken a variety of forms entailing legal, informational, and other institutional restrictions limiting the non–monetary assets that can be used in trading activities. Those models that stress the medium of exchange role of money arising from these restrictions generally fall into the class of cash in advance (CIA) models

where the working hypothesis is that all potential non–money transactions entail an arbitrarily large transaction cost and thus are never undertaken. In the real world, of course, distinctions between money and non–money transactions are not so stark and, while there may be a small subset of assets that can be used as media of exchange, there are some temporary abodes of purchasing power that can more easily be converted into media of exchange than others. An agent might then favor the holding of these securities at some pecuniary cost over other assets that are not so easily convertible to money.

A model that takes account of these differences in the marketability of assets has been developed by Feige and Parkin (1971). Their work has been put into an intertemporal context by Fried and Howitt (1983) and bears on the issue of the determination of interest rates. Briefly stated, agents wish to maximize (3.5) subject to the constraint:

5.1) $\dot{a}_t \equiv (w_t + tr_t) + i_t(a_t - m_t - b_t) + i_{bt}b_t - (\pi_t + n)a_t - e_t$

where m_t is per capita real cash balances

b_t are per capita real government bond holdings

$i_t = r_t + \pi_t$, is the nominal return on "illiquid" capital

i_{bt} is the nominal return on government bonds,

π_t is the rate of inflation, and

e_t is the gross level of consumption expenditures

Technology is the same as before and government policy consists of setting 2 policy parameters: the rate of growth of government liabilities and the mix of money to interest bearing debt. The "wrinkle" is that, to transform output into consumption goods, resources are used up in a transactions technology, $h(e,m,b)$, that depends on the agents' real holdings of money and bonds, and on the gross level of consumption expenditure. Thus

5.2) $c_t = e_t - h(e_t, m_t, b_t) = f(e_t, m_t, b_t)$

where $\ell_{me}, \ell_{be} \geq 0$, $\ell_e > 0$ and $\ell(e,m,b) \leq e \; \forall \; e,m,b$. Maximizing (3.5) subject to (5.1) and (5.2) give the optimality conditions

$$\dot{\lambda}/\lambda_t = \delta - (r_t - n),$$

$$\ell_m(e_t,m_t,b_t)/\ell_e(e_t,m_t,b_t) = i_t, \text{ and}$$

$$5.3) \quad i_{bt} = i_t - \ell_b(e_t,m_t,b_t)/\ell_e(e_t,m_t,b_t)$$

Equation (5.3) indicates that, if bonds can be used to reduce exchange costs ($\ell_b > 0$) then, like money, the yield on easily marketable securities will have a market rate of interest less than that on capital. Nonetheless it should be stressed that all securities yield their holders the same total return: Capital pays this return solely in the form of interest income while money provides this return in the form of a reduction in the cost of transforming expenditures into final consumption. Government bonds provides this return in part by market interest and in part indirectly through reduced trading costs.

In this framework measured real interest rates on bonds will depend upon the exchange technology and the availability of other inputs used in that process. In particular, the quantity of real cash balances should affect the real yield on bonds. Indeed, the work by Fried and Howitt suggests a general presumption that increases in inflation will reduce the real rate of return on government securities. The reason for this is that bonds are a substitute for money in providing exchange services; if the real return on money falls (inflation increases) agents increase their demand for other factors that can substitute for these services. Government interest bearing securities are such a substitute and thus their real rate of return decreases with increased inflation. This may provide at least a partial explanation both for the historically low real rates of return on government securities during the 1970's and for the magnitude of the equity risk premium.

5.3 Financial Intermediaries

Financial Intermediaries (FI's) also can provide assets yielding monetary services. Fried and Howitt suppose one of the roles of FI's in the economy is to transform illiquid claims to income streams into liquid claims similar to the government bond discussed above. This transformation process is not costless so that a wedge is driven between the rate of interest on illiquid capital and on the rate of return paid to primary suppliers of funds on their liquid securities. The household decision problem concerning the level of deposits to hold can be treated in the same fashion as that of government securities, and give rise to a first order condition,

$$r_{dt} = r_t - \ell_d(e_t, m_t, d_t)/\ell_e(e_t, m_t, d_t)$$

where r_d is the real rate of interest on FI deposits, d, and where government securities have been replaced by d in (5.1) and (5.2). What differs in the FI case relative to the previous analysis is that, unlike government securities, the supply of deposits is endogenous. Fried and Howitt argue that, under the assumptions of non–decreasing marginal costs to providing liquid assets and positive marginal liquidity services of high powered money to the FI, an increase in the rate of inflation will again decrease the real rate of interest on liquid assets relative to the real return on capital. Thus an endogenous supply of liquid assets remains consistent with the implication that the rising rate of inflation during the 1970's was in part responsible for the low real rates of interest on readily marketable securities observed in that period.

In the 1980's real interest rates on liquid assets have increased to historically high levels and, while inflation has decreased relative to the 1970's, it still remains at high levels relative to the post war period until the 1970's. A static theory of the production of transactions services by FI's is unable to explain this. However, in North America, there appear to have been massive changes in the technology used by FI's during the late 70's and into the 1980's. We would argue that, at least in part, this burst of technological change was induced by the high inflation combined with the regulatory environment of the 1970's. In

particular, ceilings on the nominal interest rates some financial intermediaries could pay on their liabilities combined with high nominal rates on their assets provided these institutions with an incentive to attract funds by improving the liquidity services provided their depositors. In addition, the willingness of former bank depositors to seek out alternative supplies of liquid assets from non–banking institutions provided these types of FI's an incentive to devote greater resources toward providing monetary services and, in the process, innovating in the types of financial instruments they offered. These innovations became a permanent part of the process of transforming illiquid assets into liquid ones. As such, the cost wedge between the rates at which FI's borrowed and lent funds was less in the 1980's than in earlier periods, increasing the stock of liquid securities in the economy. This, in turn, would suggest that the rates of return on all securities that provide monetary services, including government bonds, would tend to rise relative to illiquid securities.

5.4 Incomplete Markets and Uncertainty

To examine the effect of incomplete markets in an uncertain world we need to develop a model where economically meaningful trades can take place such that the absence of these trades constitutes an economic cost on agents. The representative dynamic agent paradigm suffers in this regard because no trades actually occur; the two period lived OG model also raises a problem because no one would purchase a security that cannot later be sold when the agent is old, which is the only other trading period in his lifetime. We thus use a variant of the OG model where agents live three periods. This framework provides agents an opportunity to make non–trivial economic decisions both before and after some uncertainty is resolved. There are two assets, S and L, with known, certain payoffs one and two periods in the future respectively. A unit of either security pays one unit of consumption good at maturity. There is a market for short term securities, S, that mature in one period. There is no secondary market for two period securities, L. Thus a two period discount bond issued at date t must be held to its maturity date at $t+2$.

Assume population is constant and normalize the population of each generation at *1*. Let all generations have the same set of preferences and (appropriately dated) endowments so that we can populate the economy at every date with three representative agents that differ only by their stage of life cycle. Endowments to a member of generation t are exogenous and consist of $\underline{\omega}^1_t$, $\underline{\omega}^2_{t+1}$, and $\underline{\omega}^3_{t+2}$ goods in the 3 periods of the agent's life. The agent can borrow up to the amounts $\underline{S}^1_t$ and $\underline{S}^2_{t+1}$ on the endowments $\underline{\omega}^2_{t+1}$ and $\underline{\omega}^3_{t+2}$ that will be realized in the subsequent period.[1] To keep the analysis reasonably transparent it is assumed that $\underline{S}^i_t = \underline{S}^i$ for all t, $i=1,2$. The sum, $\underline{S} \equiv \underline{S}^1 + \underline{S}^2$, will serve as an analog to the supply of one period securities in the economy. Our analog for the long term, or two period, security, L, consists of the old age endowment which cannot be borrowed one period in advance, i.e.

$$\underline{L}_t \equiv \underline{\omega}_{t+2} - \underline{S}^2_{t+1}$$

As with the representative agent model we can price this "security" at date t as if it traded at price P_{lt} in a competitive market although no such trades ever take place: P_{lt} will be the shadow price at date t that would just be sufficient to generate a zero excess demand for 2 period securities by a young member of generation t. The price of one period securities, P_s, would be an observable price in this economy as actual trades between the young and middle aged can occur.

Under these assumptions an equivalent method to describe the endowment stream to a member of generation t is:

$$(\underline{\omega}^1_t + P_{st}\underline{S}^1 + P_{lt}\underline{L}_t,\ \omega^2_{t+1} + P_{st+1}\underline{S}^2,\ 0)$$

where $\omega^2_{t+1} = \underline{\omega}^2_{t+1} + \underline{S}^1$. We assume that the only source of uncertainty in the economy is the endowment income of the middle aged. For expositional purposes suppose that it is distributed:

$$\tilde{\omega}^2_t = \omega^2(1+\eta\tilde{\varepsilon}_t)$$

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[1]The reason they cannot borrow against all their future endowment could be because of moral hazard, incentive compatibility or other asymmetric information problems that need not concern us at this juncture.

where $\tilde{\varepsilon}_t$ is i.i.d. with zero mean and whose support is such that $1 + \eta\tilde{\varepsilon}_t$ is non–negative. Further, suppose $\omega_t^1 = \omega^1$ and $\underline{L}_t = \underline{L}$ for all t.

At any date t, 3 generations participate in the economy. The old of generation $t-2$ have no economic decision to make: they simply consume their wealth available from the maturation of their short and long term securities "purchased" earlier. That is:

$$c_t^3 = S_{t-1}^2 + L_{t-2}^1.$$

where S_{t-1}^2 and L_{t-2}^1 are the amounts of S and L demanded in periods $t-1$ and $t-2$ respectively.

The middle aged may or may not face a meaningful decision problem. If they are liquidity constrained then consumption is equal to the value of their now known income, ω_t^2 plus the value of their short term assets, $S_{t-1}^1 + P_{st}\underline{S}^2$, i.e.

$$c_t^2 = \omega_t^2 + S_{t-1}^1 + P_{st}\underline{S}^2$$

If they are not so constrained then the decision problem is one of choosing a consumption plan under certainty covered in section 3, with the 2 period OG model. The relevant first order condition is that

$$P_{st} = \beta\psi(c_{t+1}^3, c_t^2)$$

and the demand for middle aged consumption is:

$$c_t^2 = c^2(P_{st}, V_t^2), \quad \partial c^2/\partial P_{st} \gtrless 0, \partial c^2/\partial V_t^2 > 0,$$

where $V_t^2 = \omega_t^2 + S_{t-1}^1 + P_{st}(\underline{S}^2 + L_{t-1}^1)$.

Thus

5.4) $$c_t^2 = \min\{c^2(P_{st}, V_t^2), V_t^2 - P_{st}L_{t-1}^1\}.$$

A young agent of generation t wishes to maximize the standard additive preference function:

5.5) $$E_t\phi_t = u(c_t^1) + E_t[\beta u(c_{t+1}^2) + \beta^2 u(c_{t+2}^3)]$$

subject to the constraints:

$$5.6)\quad \begin{cases} \omega^1 - c_t^1 + P_{st}(\underline{S}^1 - S_t^1) + P_{\ell t}(\underline{L} - L_t^1) = 0 \\ \tilde{\omega}_{t+1}^2 + S_t^1 - c_{t+1}^2 + \tilde{P}_{st+1}(\underline{S}^2 - S_{t+1}^2) = 0 \\ L_t^1 + S_{t+1}^2 - c_{t+2}^3 = 0 \end{cases}$$

$$5.7)\quad S_{t+1}^2 \geq 0$$

$$S_t^1,\ c_{t-1+i}^i \geq 0 \qquad i = 1,2,3$$

where $P_{it} \equiv \frac{1}{R_{it}}$, is the price of asset i at date t, $i = s,\ell$. For the individual agent the fundamental source of uncertainty is his income in middle age; which will impact on his consumption then, and indirectly on consumption in his old age, both from his next period savings decision and from the price of short term securities in $t+1$.

Maximizing the expected utility of a young agent gives the first order conditions.

$$5.8)\quad P_{st} = \beta E_t \psi(c_{t+1}^2, c_t^1),$$

$$5.9)\quad P_{\ell t} = \beta^2 E_t \psi(c_{t+2}^3, c_t^1) = P_{st}[\beta E_t u'(c_{t+2}^3)/E_t u(c_{t+1}^2)]$$

$$5.10)\quad E_t(u'(c_{t+1}^2)P_{st+1}) = E_t u'(c_{t+2}^3)\beta + \hat{\lambda}/\beta$$

where λ is the Lagrangian attached to the non–negativity, or liquidity, constraint, (5.7). The demand for consumption by the young can be characterized by:

$$c_t^1 = c^1(P_{st}, \hat{P}_{st+1}, V_t^1; \tilde{\varepsilon}_{t+1}),$$

where $V_t^1 = \omega^1 + P_{st}\underline{S}^1 + P_{\ell t}\underline{L}$.

Equations (5.8) and (5.9) correspond in form to (4.5) and (4.6) related to the risk free asset. Equation (5.10) on the other hand, links the liquidity constraint, (5.7), to the term structure. Rearranging (5.10) and using (5.8) and (5.9) we get :

$$5.11)\quad P_{\ell t} = P_{st}\hat{P}_{st+1} + \mathrm{cov}_t(\beta\psi(c_{t+1}^2, c_t^1), P_{st+1}) - \hat{\lambda}/u'(c_t^1).$$

If the liquidity constraint (5.7) is not binding in any state then (5.11) reduces to (4.8); if there are some states of nature where (5.7) is expected to hold with equality then the price of the long term bond will be less than a comparable long term security that is marketable. The reason for this is clear: In states where the middle aged agent is constrained the marginal utility of c^2 is higher and the marginal utility of c^3 lower than would be the case if the agent was free to borrow. Thus total utility is reduced due to the inability to sell seasoned long term securities at the expected price of a one period security in middle age.

Knowing the behavior of the young is sufficient to obtain information on the implicit price of the long term security *given* the behavior of the one period bill. To determine the behavior of short term rates we must impose the market clearing conditions:

$$Y_t = c_t^1 + c_t^2 + c_t^3,$$

$$S_t^1 + S_t^2 = \underline{S}^1 + \underline{S}^2 \equiv \underline{S},$$

and

$$L_t^1 = \underline{L}.$$

where

$$Y_t = \omega_t^1 + \tilde{\omega}_t^2 + \underline{S}_{t-1}^1 + \underline{S}_{t-1}^2 + \underline{L}_{t-2} = \omega^1 + \tilde{\omega}_t^2 + \underline{S}^1 + \underline{S}^2 + \underline{L}$$

is total income in period t.

Closed form solutions are notoriously difficult to obtain in models of this sort.[2] We shall suppose a rational expectations equilibrium exists and that P_{st+i} satisfies the difference equation (5.8). In what follows we discuss the behavior of asset prices in this stochastic equilibrium.

Suppose first that the middle aged are liquidity constrained. Then the goods market clearing condition can be reduced to:

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[2]Zeldes (1989a) has had some success at providing approximations using numerical methods for a model similar to the one developed here.

5.12) $c^1(P_{st}, \hat{P}_{st+1}, V_t^1; \varepsilon_{t+1}) = \omega_1 - P_{st}S_t^2$

Assume a unique rational expectations equilibrium exists for this problem with $P_{st} = P_s^*$ for all t in which (5.7) holds, and that (5.12) is such that $dS_t^1/dP_{st} < 0$. Next consider states where (5.7) is not binding. From the separability of preferences, $0 < \partial c^2/\partial V^2 < 1$, so that $\partial S_t^2/\partial \omega_t^2 > 0$. Therefore increases in ε_t give rise to an increased demand for S_t^2 and, since $dS_t^1/dP_{st} < 0$, an increase in their price. From this we can conclude that P_s^* is the minimum equilibrium price and that

$$\mathrm{cov}_t(P_{st+1}, \beta\psi(c_{t+1}^2, c_t)) < 0$$

so long as there are some states in which the middle aged are not constrained by (5.7). In turn, this means that 2 period assets will sell at a discount to the expected price for the same payout of a sequence of two one period assets according to (5.11).

An increase in the mean preserving spread will, with $u''' > 0$, decrease the risk free rate of interest, R_{st}. It will have an ambiguous effect on the two period rate: It will decrease R_{tt} insofar as R_{st} and $\hat{R}_{st+1}$ fall but will increase the risk premium, generating the same ambiguity as noted in section 4; and it will increase the liquidity premium, further increasing the spread between the risk free asset and the 2 period bond.

Despite the fact that $E(\varepsilon_t, \varepsilon_{t+1}) = 0$, the expected price of one period assets next period will depend upon this period's endowment shock. This is because the current endowment shock will affect the distribution of assets next period. Because the propensities to spend out of wealth differ across age groups this redistribution will alter consumption and asset demands next period. To see this suppose that there is a positive endowment shock this period so that $\omega_t^2 > \omega^2$. From our previous analysis this means that $S_t^1 > S_t^1(0)$ and $P_{st} > P_{st}(0)$, where the 0 in parenthesis indicates the value that would have occurred if $\varepsilon = 0$ in period t. Thus, the middle aged next period will have less wealth than would have occurred had $\varepsilon_t = 0$.

Therefore, from (5.4), the demand S^2_{t+1} and P_{st+1} will be less than otherwise would have been the case if the middle aged are not rationed.[3] Thus prices will be negatively serially correlated even though the fundamental disturbance is serially independent.[4]

5.5 Futures Markets

We now want to examine the effects of the liquidity constraint on asset prices for a given distribution of $\tilde{\varepsilon}_t$. To do so it is convenient to first establish a new market that can be thought of either as an insurance or as a futures market. This will permit us to distinguish between the effects of risk and marketability of securities on asset prices. In particular, suppose that a new market is established in this liquidity constrained environment where agents can place bets on the price of one period securities one period hence. A "long" bet placed in period t receive the amount $P_{st+1} - P_{ft}$ in period $t+1$ once the price P_{st+1} becomes known; a short bet pays that amount. P_{ft} is competitively determined so that the amount of long and short bets are equal.

Let F^1_t and F^2_t denote the number of long bets made in period t by a member of the young and middle aged generation respectively, and treat short bets as negative long bets. Because the payoffs are made at $t+1$ the old at time t cannot place bets. However the payoffs at $t+1$ are made between the then middle aged and old (of generation t and $t-1$ respectively). For the young the following budget constraints replace (5.6):

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[3]If the middle-aged are constrained by (5.7) then P_{t+1} will be the same as with any other constrained equilibrium. This is because the propensity to spend is equal to one for both the middle aged and old in this case. Note also that the lower is S^1_t, the greater is the ε_{t+1} at which the liquidity constraint becomes operative. This also means the liquidity discount, $\hat{\lambda}/u'(c_t)$ depends negatively on P_{st}.

[4]For more on these distribution effects, see Huffman (1987).

$$5.13) \quad \begin{cases} \omega_t^1 - c_t^1 + P_{st}(\underline{S}^1 - S_t^1) + P_{Lt}(\underline{L} - L_t^1) = 0 \\ \tilde{\omega}^2 + S_t^1 + (\tilde{P}_{st+1} - P_{ft})F_t^1 - c_t^2 + \tilde{P}_{st+1}(\underline{S}^2 - S_{t+1}^2) = 0 \\ S_{t+1}^2 + L_{t+1}^1 + (\tilde{P}_{st+2} - \tilde{P}_{ft+1})F_{t+1}^2 - c_{t+2}^3 = 0. \end{cases}$$

Maximizing (5.5) subject to (5.7) and (5.13) gives first order conditions (5.8), (5.9), (5.10) plus

$$5.14) \quad E_t[\beta u'(c_{t+1}^2)(\tilde{P}_{st+1} - P_{ft})] = 0$$

and

$$E_t[\beta^2 u'(c_{t+2}^3)(\tilde{P}_{st+2} - \tilde{P}_{ft+1})] = 0$$

Using (5.8), (5.14) can be rewritten as

$$5.15) \quad P_{ft} = \hat{P}_{st+1} + \text{cov}_t(\beta\psi(c_{t+1}^2, c_t^1), P_{st+1})/P_{st}.$$

For the middle aged the additional first order condition is

$$5.16) \quad E_t(\beta u'(c_{t+1}^3)(\tilde{P}_{st+1} - P_{ft}))$$

$$= P_{st}(\hat{P}_{st+1} - P_{ft}) + \text{cov}_t(\beta\psi(c_{t+1}^3, c_t^2), \tilde{P}_{st+1}) = 0.$$

The additional market clearing condition needed for the "futures market" is that

$$F_t^1 + F_t^2 = 0.$$

The futures market will be viable; that is, $F_t^i \neq 0$ for at least some t. To see this note that, from (5.15), the young are willing to take a bet with an expected loss if, by doing so, it reduces the volatility of future consumption. Indeed, because $u'(c_{t+1}^2)$ is negatively correlated with P_{st+1}, the young would be willing to take a short ($F^1 < 0$) position in the futures market up to the point where the futures price discount, $\hat{P}_{st+1} - P_{ft}$, is equal to $-\text{cov}_t(\beta\,\psi(c_{t+1}^2, c_t^1), P_{st+1})/P_{st}$. In this manner they can hedge some of the risk associated with their middle–aged income levels: when ω_t^2 is low $P_{st+1} - P_{ft}$ is negative, compensating for some of the decreased income. Given that the young are willing to pay a positive expected price,

$(\hat{P}_{st+1} - P_{ft}) = -\text{cov}(\beta\, \psi(c^2_{t+1}, c^1_t),\, P_{st+1})/P_{st}$ to bet in the futures market, condition (5.16) indicates that the middle–aged would be foolish not to take advantage of the opportunity to bet against them.[5] Thus the middle aged will take long positions in the futures market.

What the futures market does, in effect, is to reduce the variability of income to the representative agent over his entire lifetime. It does so by permitting diversification over time: agents are able to diversify some of the uncertainty of middle age by accepting some risk in their old age. Because ε_t and ε_{t+1} are not perfectly correlated, the overall risk to any agent over his lifetime is reduced.

This has implications for asset prices. In particular, the decreased lifetime risk perceived by the individual acts in the same manner as a decrease in the mean preserving spread on the risky "security" having an expected payoff ω^2. As such the risk free security S, will have an increase in its rate of return. Similarly, the price of L_t will fall on this account. However, there are offsetting factors here. One such factor is that, because total income in middle age is no longer as volatile, the number of states in which (5.7) is binding is now reduced. Thus the relative value of liquidity is reduced, increasing the value of the illiquid asset. We can also conclude that the conditional variability of asset prices will decrease with the introduction of the futures market relative to the absence of this and the secondary markets. This follows immediately from the effective intertemporal smoothing of ω^2_t once futures transactions are included. This smoothing implies that $\psi(\tilde{c}^2, c^1)$ is less volatile and therefore, so too is P_s.

One aspect of this equilibrium is that the young go short in the futures market while at the same time, they (are constrained to) hold two period securities. In effect they have imperfectly "covered" a portion of their long term securities to the amount $-F^1$. That is, if the young held 1 unit of L and sold 1 futures contract, then in middle age, he would have one unit

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[5] If the middle aged did not enter the market, then $\text{cov}(\beta\, \psi(c^3_{t+1}, c^2_t),\, \tilde{P}_{st+1}) = 0$ violating the first order condition (5.16).

of an (unmarketable) one period asset in the form of a seasoned unit of L and a contract to supply a one period security whose price was previously negotiated at the price P_{ft-1}. Suppose for a moment that the agent could use the seasoned asset to meet the implicit obligation of his futures contract at the price P_{st}. Then, in effect, when young, the combination of L and $F^1 < 0$ means the conversion of $-F^{-1}$ long term securities into marketable 2 period securities. If the agent was in a position where he wished to sell securities for consumption, then he has effectively done so; if he did not wish to he would use the proceeds from the futures contract, P_{ft}, to purchase a security at price P_{st+1}, pocketing the difference or selling some of his endowment ω_{2t+1} to make up any loss. This would leave him in precisely the same position as if he retained his long term security. (Indeed, he purchases the security from the old who had the opposite side of the contract and who wishes to sell it for consumption goods.) But, of course, in some states he cannot sell the security because (5.7) is binding.

Suppose that the liquidity constraint was never binding (e.g. there was a compensating change, $\Delta L_t = -\Delta S^2_{t+1} < 0$, such that $S^2_{t+1} > 0$ for all possible states of nature). Then $\lambda = 0$ and the long term security would be priced at

$$P_{\ell t} = P_{st} P_{ft}.$$

Indeed it can be shown that, if (5.7) is never binding, then the existence of a futures market is equivalent to an economic environment where capital markets are perfect, i.e., a secondary market for two period bonds exist. Thus the remarks comparing the behavior of asset prices with and without a futures market provides an initial look at the effects of incomplete markets on asset prices relative to the standard paradigm of complete markets.[6]

5.6 Secondary Markets

To continue the analysis, it remains to compare an environment with complete markets with one having a futures market, no secondary market for L, and (5.7) binding in some states

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[6]It is entirely possible that for a given distribution of ε, (5.7) is binding in some states when no futures markets exist whereas, with futures markets, agents are sufficiently diversified so that (5.7) is never

of nature. We shall argue that the futures market provides one characteristic of assets, namely the ability to diversify risk over time and thereby to partially insure oneself against aggregate risk. Perfect capital markets provide, in addition, flexibility which provides the ability to "self insure" to a greater extent than available with futures markets, and to take advantage of opportunities once the previous uncertainty is resolved. More prosaically, futures markets insure against *ex ante* risk; perfect capital markets also provide the flexibility to act once the actual outcome is known. This additional flexibility will be reflected in asset returns.

In particular, under the assumptions so far made, the increase in flexibility will increase the volatility of rates of return and increase the return on short term securities relative to asset returns when futures markets exist. To see this note that, at some low value of ε, ε^0, below which agents are liquidity constrained, the price, $P_s(\varepsilon^i) = P_s^*$ for all $\varepsilon^i \leq \varepsilon^0$. Now suppose agents are not constrained by (5.7). Then, if $\varepsilon_t < \varepsilon^0$ they would chose to sell more short term securities (i.e. unload some of their seasoned two period assets). This would imply that $P_s(\varepsilon^i) < P_s^*$, and, since P_s^* is at the low extreme of the price distribution, this must necessarily cause an increase in the measured movements in asset prices. Furthermore, the return on the risk free security will rise: From (4.5), $R_t^f = u'(c_t)/\beta E_t u'(c_{t+1})$ and, for the young, $E_t(u'(c_{t+1}^2))$ falls since agents can now better allocate their resources in middle age. Thus their demands for one period assets falls. For the middle aged, their demand function in previously unconstrained states is unchanged, while in previously constrained states they now plan to *sell more* one period securities (in the form of seasoned 2 period bonds) than they would have been able to sell before.[7]

We are now in a position to compare returns when assets are and are not marketable. Marketable long term securities will increase the rate of return on one period securities relative

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binding.

[7]A further way to indicate that $E(P_{st})$ will fall is to note that the total stock of S_t has increased by $\underline{L}_{t-1}^1$ to $\underline{S}^1 + \underline{S}^2 + \underline{L}_{t-1}^1$ since the seasoned 2 period assets now can trade as perfect substitutes for one periods bonds.

to the return when two period securities are not marketable. It does this for two reasons. First it will increase because the existence of marketable long term securities reduces risk faced by the agent by permitting diversification as with the existence of the futures market. Second the return is further increased because other marketable securities provides the flexibility of action that is an attractive characteristic of short term securities. This increase in substitutes for flexibility decreases the demand for flexibility from one period securities, increasing their average return. We cannot say whether the volatility of rates will increase or decrease with the existence of other marketable securities since, through their insurance function, overall price volatility is decreased, but the additional flexibility causes measured prices to fluctuate more, even though it causes marginal utility to fluctuate less.

5.7 Non–Representative Agents and Uncertainty

Agents with identical preferences who populate an economy with complete contingent claims markets need concern themselves only with aggregate risk. As in the Blanchard model individual specific risk can be insured against. If markets are incomplete or too costly to operate this might no longer be true. The relevant risk to individual decision–making is not only aggregate risk but also that part of the specific risk that the individual cannot insure against. It is this risk that will also be relevant in determining equilibrium asset prices. As a result, artificial economy models calibrated to aggregate movements of output and consumption will not likely be able to explain relative asset prices simply because the aggregate risk measure is a seriously downward–biased estimate of what the individual agents perceive the risks to be. Mankiw (1986) makes use of this argument to explain the equity premium puzzle posed by Mehra and Prescott. In a simple model he shows that, if all agents are *ex ante* identical, but endowment shocks are concentrated on a randomly selected subset of agents then, if $u''' > 0$, the smaller the proportion of the population affected by the shock the greater will be the risk premium. This resolution of the equity premium puzzle is in many

ways similar to that of Rietz. What matters is the size of the possible shock to the individual: Rietz increases this risk by supposing a low probability state with a large negative shock to the economy; Mankiw achieves the same result by concentrating a shock of given size on a small subset of the population.

The aggregate data also conceals the relevance of liquidity constrained behavior by large groups in the population. While there has been some findings of "excess sensitivity" of consumption indicating the possibility of borrowing constrained behavior (Flavin (1981), Hall and Mishkin (1982)) there are others that contend that consumption is not sensitive enough (c.f. Deaton, Campbell and Deaton). However, Zeldes (1989b) uses panel data partitioned on the basis of asset to income ratios of families. With it he finds that life cycle consumption behavior was consistent for those families with relatively large amounts of liquid assets but that the behavior of those with few assets appeared to violate the Euler conditions and were excessively sensitive to income changes. Zeldes reports on other work with Mankiw that partitioned the sample on the basis of stock ownership. Consumption behavior of stock owners and non stock holders is reported to be significantly different. This too may provide a partial explanation for the equity premium puzzle.

5.8 Summary and Implications

To provide a reasonable explanation of the magnitude of the equity premium we have considered the possibility of costly to operate markets. There are two mechanisms that can be used to help explain this premium. The first of these is that costs of buying and selling assets in some markets is less than in others. The assets traded in the low cost, liquid, markets can be viewed as partial money substitutes. Thus part of the total rate of return accrues in the form of a reduction in the costs of exchange in goods markets. Since the total return on all assets in a certain world must be equal it means that the measured rate of interest on liquid assets will be below that on illiquid assets which cannot be used to reduce the costs of

exchange in other markets. We would argue that government debt as well as many of the liabilities of financial intermediaries can be regarded as playing a positive role in reducing the costs of exchange and will thus have lower market rates than capital. One implication of this approach is that increases in expected inflation will generally decrease the measured real rates of return on liquid assets that are either exogenously or endogenously supplied by governments or financial intermediaries respectively.

The second mechanism by which incomplete markets increase the equity premium is by not permitting agents to completely diversify their individual specific risk. One case that we consider does not permit diversification over one's lifetime. Here we showed that the absence of secondary markets for income streams or, more generally, the existence of borrowing constraints, decreases the rate of interest on securities that can be freely traded relative to those that lack established markets. We also note that the effect of incomplete markets on asset price volatility can be decomposed into two parts. To the extent that agents cannot use an asset to insure against *ex ante* risk, the volatility of asset prices will increase. To the extent that the agent cannot use an asset to respond to risk ex post (i.e. he cannot use an asset for flexibility once an outcome is known) it decreases the price volatility of other assets. The second case of incomplete diversification involves the absence of intragenerational insurance: Because agents in similar circumstances are unable to insure against individual specific risk, a possible aggregate shock of given size but concentrated on a subset of the population will understate the perceived risk of that event to the individual. Because asset prices are based on the risk perceived by individuals, agents would wish to pay more for a riskless security relative to a risky one than would be indicated by treating aggregate risk as the perceived risk to the individual.

Incomplete markets may provide a partial explanation for the behavior of interest rates on liquid assets over the last two decades. The increase in inflation in the 1970's increased the attractiveness of money substitutes relative to cash causing their real yields to fall during that

period. The burst of financial innovation that also occurred toward the end of that decade might be regarded as a permanent effect of that inflation when combined with the regulatory constraints then in place. With the removal of the more onerous of these regulations and the decrease in inflation that occurred in the 1980's financial intermediaries found that they could provide liquidity services more cheaply than before, decreasing the spread between the rates paid on their assets and liabilities and increasing the relative supply of liquid assets held by primary lenders. Because the supply of money substitutes increased, the marginal liquidity services of these assets fell, bringing the measured rate of interest on these securities more in line with those on assets that provided no liquidity services. Furthermore the new financial instruments developed in this period suggests that people were better able to diversify their intertemporal risks also increasing the rate of interest on marketable assets.

Increases in interest rates arising from innovations that reduce the costs of exchange or of financial intermediation should not be viewed as undesirable or, somehow, an indication of undesirable trends in the economy. What we observe is the increase in rates of interest charged to existing borrowers or paid to lenders. What we do not observe is the decrease in the effective cost of funds to new users of financial markets when many of the hidden costs of obtaining funds—collateral costs, monitoring, rationing etc.—have been removed. What you see is not necessarily what you get!

6 PRODUCTION, INVESTMENT AND THE RATE OF INTEREST

6.1 Introduction

In the previous sections we analyzed the consumers decision and its effect on interest rates. In this section we examine one extension of the household — the firm — and its influence. We suppose the firm operates by transforming inputs into outputs so as to maximize the value of the firm and household wealth. It is assumed that firms operate in a competitive environment. This applies not only for the input and output markets, but also in the market for financial claims on the firm.

There will be three issues that will concern us. The first is the question of how we model the capital investment decision of the firm. For this we shall retain the framework associated with Tobin (1969) of a demand and supply price of capital. The demand price for an investment project is the valuation of the project in accord with the preferences and precepts laid out in the preceding three sections: Households value a project based on the project's risk, expected return, and covariation with consumption to determine a price the household would pay to hold title to the income flowing from the specific project.[1] The suppl price is the relative price of capital goods used in the project measured in terms of consumption goods if the capital stock per worker were not growing. It is sometimes useful to normalize this price to equal one. This is convenient for integrating the analysis into the neoclassical one sector growth model discussed in section 2. In more complex models the relative price of capital goods will itself depend on the aggregate capital labor ratio so that, as defined above, it will change during the transition to the steady state. It should also be noted that at each point of time, the price charged by suppliers will always equal the demand price investors are willing to pay by the very nature of an equilibrium. Most innovations in this area

[1]This will correspond to the marginal "q" of the firm.

relate to the nature of the adjustment costs separating the marginal product of installed capital from the marginal efficiency of investment.

The second area that will concern us is the effect of government tax and spending policies on the marginal efficiency of investment. Capital taxation is a sufficiently complex matter that, in the interest of manageability, we shall only briefly touch upon it. We make two points: First, what is important to the equilibrium level of the capital stock is the total tax rate on capital investment although the measured interest rate will depend upon whether this tax is levied on households or firms. Second, the demand for capital is probably more price elastic than other factors of production. As a result, the welfare costs are greater if capital is taxed instead of labor. Government spending on social capital represents an input into the production process and, as such, will affect the rate of interest and the marginal product of capital.

The third issue that will concern us is the nature of the external shocks that impact on the economic environment through the firm's decisions. These have often been treated as shocks to the firm's production technology or as technological innovations. We add a third interpretation which will become important later when we examine the open economy, namely changes in prices of intermediate goods that are given exogenously to the economy.

6.2 Investment Under Certainty

The criterion used to choose among projects and to determine which ones to invest in at any date t is the present value criterion: rank projects by their net present value and invest in all those projects for which the net present value is non–negative. Elements that enter this decision include the cash flow from a project, the structure of the tax system, the required rate of return demanded by households and the cost of the project.

Consider a firm at time t=0 faced with the opportunity to purchase any of $j \in \{1,...J\}$ possible investment projects. Each project is characterized by a sequence of net cash flows,

$\{\theta_t^j\}_{t=1}^{\infty}$, of taxes levied on that flow, $\{\tau_{ft}^j\theta_t^j\}_{t=1}^{\infty}$, of economic depreciation, $\{\delta_t^* P_{k0}^j K_0^j\}_{t=1}^{\infty}$, and a cost for the project, $P_{k0}^j K_0^j$, where

$$\theta_t^j = Y_t^j - w_t N_t^j - P_{mt} M_t^j - \delta_t^* P_{k0}^j K_0^j,$$

Y_t^j is output produced from project j at t using inputs labor, N_t^j, and intermediate goods, M_t^j, selling at relative prices w_t and P_{mt} respectively, K_t^j is an index of capital purchased in the project, P_{k0}^j is the unit price of purchasing and installing a unit of that capital discounted to the present, and τ_{ft}^j is the (ex post) tax rate on the net cash flow that are paid by the firm.

The present value of any project, j, is defined as

$$q_t^j K_t^j \equiv \sum_{i=1}^{\infty} \{[\theta_{t+i}^j (1 - \tau_{ft+i}^j) + \delta_{t+i}^* P_{kt}^j K_t^j] / \prod_{h=1}^{i} R_{t+h-1}\}$$

where q_t^j is the demand price of project j at date t. The present value criterion states that all projects should be undertaken for which $q_t^j K_t^j \geq P_{kt}^j K_t^j$, or

$$q_t^j \geq P_{kt}^j$$

A number of elements in the present value formula need amplification. One element, $\{\tau_{ft}^j\}_{t=1}^{\infty}$, masks an enormous amount of complexity. The tax rates levied on a given project will depend not only on economic profits, but also on accounting conventions associated with different types of capital, including inventories, and their "official" depreciation rates, the rate of inflation (to the extent that it is not properly indexed), the existence of investment tax credits or other business subsidies, and whether investors are repaid in the form of interest, dividends, or capital gains. We wish to abstract from those issues.[2] Instead we shall suppose that for each dollar of capital income, the household ultimately receives $(1 - \tau_k)$ dollars, where τ_k is the tax rate on capital. This tax, in turn is composed of two elements, τ, which is the

[2]For a comprehensive study of the effects of alternative capital taxation schemes on the allocation of resources, see Sinn (1987). See also Gordon and Wilson (1989).

common rate paid on all forms of income received by households and paid directly by them, and τ_f which is the average rate paid directly by firms (and thus indirectly by households) given that firms have arranged their investment policy and financial structure "optimally" in light of the various tax and subsidy policies imposed by the government, such that

$$\tau_k = 1 - (1 - \tau)(1 - \tau_f).$$

It may be useful to think of τ corresponding to the income tax rate and τ_f to the corporate tax rate even though the complexities of the tax law makes a mockery of such an analogy.

The second element that presents some conceptual problems is the price of capital goods, P^j_{kt}. This is usually focused upon to distinguish between the marginal efficiency of capital and investment. One modeling strategy is to suppose that capital can be installed in one period and the economy consists of two sectors, a consumption goods sector and a capital goods sector. The production technology in the two sectors are assumed to be different. Suppose that, for some reason, there is a decrease in interest rates, raising q^j_t, for all j. As more projects now become profitable, investment demand increases, increasing the relative price of newly produced capital goods and choking off some of the investment that would otherwise have been undertaken. In the subsequent period, those projects initiated in the previous period no longer require new capital goods, so the price of capital goods falls, inducing additional investment expenditure. In such a way a given change in the rate of interest at t generates a flow of investment expenditure over subsequent periods. The concept of the "supply price of capital" is a construct that permits the economist to distinguish between periods of balanced ($\dot{k} = 0$) and unbalanced ($\dot{k} \neq 0$) growth. The supply price of capital is that price of capital goods at which there is no change in the capital labor ratio. Thus, the rate of interest necessary to maintain this price will equal the marginal product of installed capital. If the rate of interest is other than this amount, the rate will not be equal to the marginal product of installed capital but it will still be equal to that on newly produced capital as the price of capital will adjust to bring about this equality.

A second method to distinguish between the marginal efficiencies of installed capital and new investment is to keep the one sector framework but to suppose that there are costs (measured in terms of foregone consumption goods) of installation where the marginal cost of installation is increasing in the level of investment. This provides the same qualitative effect as the two sector model. A third method, that has been used with some success by Kydland and Prescott (1982), is to suppose a time to build technology that permits less flexibility in choosing the timing of investment than the adjustment costs framework but does require that an investment project be required to be completed in more than one period.[3] As near as we ca tell, the implications of this latter approach are qualitatively the same as the previously mentioned methods: All indicate that, beginning with a given capital stock, the transition to the steady state will be less rapid than suggested in the neoclassical growth model developed in section 2.

6.3 Taxation

Government taxation of capital also alters the equilibrium characteristics of the steady state. Dynastic agents faced with a tax on capital will view the constraint they operate under as

6.1) $\dot{a}_t = (1 - \tau)(a_t r_t + \omega_t) - c_t - n a_t$

Maximizing the relevant Hamiltonian gives the laws of motion of (6.1) and

6.2) $\dot{\lambda}_t/\lambda_t = -[(1 - \tau)r_t - n - \delta]$.

Stating (6.2) in terms of $\dot{c}/c$ gives

6.3) $\dot{c}_t/c_t = -(u'/c_t u'')[r_t(1 - \tau) - n - \delta]$

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[3]Vintage capital models represent an additional method to view the investment decision. It could be treated in any of the three ways described here and can be characterized by the present value approach with each j describing a specific vintage. Use of a vintage model and a production technology, $y = F(N; K^1,...K^j...K^J)$ avoids the problem of constructing an index of the capital stock. On the other hand, for expository purposes, such an index is extremely useful and has been assumed throughout this essay. For additional discussion of modeling the investment decision, c.f. Abel (1980, 1985), Huffman (1985b), McDonald and Siegal (1986), and Sargent (1980).

Further, firms will maximize profits when the condition,

6.4) $(1 - \tau_f) f'(k) = r_t$

is met. Thus the steady state equilibrium condition is:

6.5) $f'(k) = (\delta + n)/(1 - \tau)(1 - \tau_f) = (\delta + n)/(1 - \iota_k)$

which will be greater than $\delta + n$ if net capital taxation, τ_k, is positive.

Suppose that instead of a tax on capital income there was a consumption tax τ_c. In this case, interpret c_t as consumption expenditure gross of taxes paid so that actual consumption, c_t^*, is

$$c_t^* = c_t(1 - \tau_c)$$

The instantaneous utility function is thus $u(c_t^*) = u(c_t(1 - \tau_c))$. Suppose further that $\tau = \tau_f = 0$. Then maximization gives:

$$\dot{c}_t^*/c_t^* = -[u'/c_t(1 - \tau_c)^2 u'']\,[r_t - n - \delta]$$

in place of (6.3). For the dynastic agent model this generates a higher level of the steady state capital stock than in the situation where capital income is taxed. In fact, in terms of the interest rate configuration in the steady state, if labor is supplied inelastically, the consumption tax is the equivalent of a lump sum tax.[4]

There is one further point regarding interest rates and capital taxation that is worth remarking on. Suppose we conduct the conceptual experiment of decreasing the income tax rate on capital (leaving unchanged the tax rate on labor) and increasing indirect taxes on the firm, leaving τ_k unchanged. From equation (6.5) it is clear that the marginal product of capital, and thus the capital stock, will be unaffected. On the other hand, as is clear from (6.3) and (6.4), the measured rate of interest will decrease. What is occurring, of course, is that the total "tax wedge" is being shifted toward the margin between the firm and the capital market

[4]It should be noted that the use of R.R.S.P.'s qualitatively converts the income tax into an expenditure tax. See Hood (1982).

and away from the margin between household and this market.[5] Furthermore, there are numerous "capital projects" that are undertaken by the household itself such as homes, cars, videos and jewelry that escape entirely the "corporate tax", τ_f. Thus a result of this experiment would imply that these durable goods would increase in household portfolios at the expense of productive capital.

6.4 Intermediate Goods

In section 4 we argued that the uncertainty connected with the oil shock of 1973 would tend to decrease the risk free rate of interest. No mention was made about the impact of the change in energy prices on the cost of capital itself. Here we want to return to the question of the energy shock and explore its effect on production and the cost of capital. We choose to do this in the context of certain environment to more clearly distinguish between the effects of uncertainty and the effects of the change in price of intermediate goods on production and interest rates themselves.

As should now be clear, in a dynastic agent paradigm an energy price shock would have no effect upon steady state interest rates, which are determined solely by the rate of time preference and the effective rate of population growth. We therefore examine this issue using an overlapping generations model. To attempt to get a handle on the stylized events of 1973–74 we shall suppose that energy is costlessly produced by a non–competitive industry controlled by "The Cartel" that arbitrarily fixes the relative price of energy at $\bar{p}_m$ and that the amount of energy supplied is demand determined. Furthermore, the proceeds from the sale of

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[5]Lest anyone think this is especially desirable it should be stressed that, with the current state of tax policy, it seems reasonable to suppose that corporate tax policy rules regarding depreciation allowances, regional and other subsidy policies etc. are likely to be much more distortionary across types of capital and across firms than the distortions caused by taxes on individuals. It is not at all clear to us that a move toward greater tax loads on firms will lead to a more appropriate allocation of capital.

energy, $p_{mt} M_t$ are used entirely for consumption by the cartel owners, who neither work nor own capital.[6]

Competitive firms produce a consumption/capital good with the technology:

6.6) $y_t = F(K_t, M_t, N_t)/N_t = f(k_t, m_t)$

where M_t is total energy supplied and m_t is energy per worker. The first order conditions for profit maximization are:

$$\text{6.7)} \quad \begin{cases} r_{t-1} = f_n(k_t, m_t) \\ \bar{p}_{mt} = f_m(k_t, m_t), \text{ and} \\ w_t = y_t - k_t f_k - m_t f_m \end{cases}$$

Because the Cartel members consume all of the proceeds from the sale of energy, their behavior will not affect the level of the capital stock, which will be determined by:

6.8) $k_t = w_{t-1} - c(w_{t-1}, r_{t-1})$

Equations (6.7) and (6.8) contain the four endogenous variables k_t, m_t, r_{t-1}, and w_t as functions of the exogenous variables $\bar{p}_m$ and w_{t-1}. If the economy is dynamically stable it will eventually converge to a steady state in which these endogenous variables attain constant values.

Suppose that the economy is initially in a dynamically efficient steady state equilibrium in which all four variables are constant over time and $r > n$, where n is the exogenous rate of labor force growth. Now consider the impact of an increase in the world relative price of energy on the equilibrium real interest rate. Differentiating the above system totally with respect to a change in p_m and solving for the change in r, we obtain:

$$dr/dp_m = \{f_{km} - (1 - c_w)m\,\Delta\}/\nabla$$

where $\Delta = f_{kk} f_{mm} - f_{km}^2 > 0$ and $\nabla = [(1 - c_w)k + c_r]\,\Delta + f_{mm}$.

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[6]This is analogous to the government running the cartel, levying a tax on young and old, and using the proceeds of the tax to purchase government goods that give no benefit to members of society.

It is easy to show that $\nabla < 0$ if the model is dynamically stable. Therefore an increase in the world relative price of energy may raise or lower the long run real interest rate depending upon whether the numerator is negative or positive. If $f_{km} \leq 0$ then $dr/dp_m > 0$, but if the technology is Cobb–Douglas then $f_{km} > 0$ and the numerator may be negative, so $dr/dp < 0$. Hence the model yields no clear prediction about the long run impact of energy price changes on the real interest rate. In order for there to be a negative relationship between r and p_m it is necessary for f_{km} to be large.

The ambiguous response of the real interest rate to changes in the price of intermediate inputs like energy is not surprising. The real interest rate is determined by productivity and thrift. There is a presumption that an increase in p_m will reduce the marginal productivity of capital, putting downward pressure on r. At the same time, it reduces the real wage, and thus the supply of savings, putting upward pressure on r. The net impact on r depends on the relative strength of these two forces.

One reason the above analysis might provide a presumption that the cost of capital, r, will rise is because of our assumption that energy producers do not choose to purchase capital with the proceeds of their energy revenue. This is the equivalent of asserting that all energy revenue goes to the old (with a marginal propensity to spend of one) in an OG model. If instead it went to the young, then the endowment of the young would be:

$$\omega_t = (w_t + \bar{p}_m m_t) = y_t - f_k k_t$$

Thus, at any w_{t-1} and $\bar{p}_m$ the level of saving would be greater and the steady state capital stock larger. An increase in the price of energy could still increase or decrease the rate of interest, but to the extent that the workers share of output, including the revenue from energy, increased, savings would be greater and the rate of interest lower. We would therefore argue that, if the oil shocks affected interest rates their impact was primarily on agents' perceptions of uncertainty and not necessarily because of the technological shifts arising from changed relative prices.

6.5 Uncertainty and Production

Introducing uncertainty into the analysis provides a richer environment but does not alter many of the qualitative results of the certainty case. The use of the present value formulation carries over to uncertainty with the qualification that the demand price, q_t^ℓ, for the ℓ^{th} project is the stream of random cash flow $\{\tilde{\theta}_t^\ell + \delta_t^* P_k^\ell K^\ell\}$ discounted by the appropriate risky rate of return provided in the earlier sections. Taxes will continue to "matter" in determining the steady state rate of interest and in the choice of the optimal capital structure. Adjustment costs will continue to generate investment over extended periods. The positively sloped supply of capital goods schedule will divide the uncertainty into price and quantity uncertainty corresponding to the land and labor models discussed earlier.

It has been argued that one aspect of production under uncertainty that bears on the determination of interest rates is that the production sector is the principal source of the stochastic shocks that impact on the economy. These take the form of technology, or productivity shocks which are the assumed impulses in recent work on real business cycles. Whether the shocks are initiated in this sector, or occur from shifts in tastes, the distribution of endowments, government fiscal and monetary policies or, in Canada's case, arise in the external sector, they impact most immediately on profits of the firm rather than factor payments to labor.

One possible explanation why profits are more volatile than either total output or labor income may relate to incomplete markets. In particular, the literature on implicit labor contracts argues that such contracts arise because of the absence of insurance markets and differences in risk preferences across agents. Briefly, the argument is that firms, or their owners, are less risk averse than workers. As such, firms find it profitable to provide limited insurance against unfavorable states of nature to their workers. This takes the form of offering fixed (real) wage contracts with some probability of layoff to their workers instead of variable wage contracts. In good states of nature, then, workers are paid less than their marginal value

product and, in bad states, more than their marginal value product. Thus the firm's profits are greater in good states and lower in bad states than they otherwise would be if auction markets were the dominant form of market structure for labor.

While this does not bear directly on interest rates it is suggestive for a number of points that we have discussed earlier. First, as discussed in section 5, the absence of complete markets will imply that there are likely to be liquidity premiums embedded in a number of asset prices. Secondly, it suggests that it may be unrealistic to suppose homogeneous attitudes toward risk represents a reasonable approach to modeling behavior. In this case the firm (and its owners) must view risk differently than its workers if such implicit contracts are to be made.[7] Third, the structure of implicit contracts is such that shocks to the system are concentrated. Not only are they concentrated on firms but also on that portion of the of the labor force that is laid off in bad states. Given that equity ownership is not widespread (Zeldes (1989b)) it suggests that Mankiw's (1986) explanation of the equity premium puzzle deserves very serious consideration.

6.6 Summary and Conclusions

In this section we have briefly described the investment decision of the firm, the effect of taxation on interest rates and some of the implications of uncertainty and incomplete markets on the organization of the firm. The first of these items is included in this essay to more fully describe the market for loanable funds. Using a present value formulation the firm can calculate a demand price for each of the possible investment projects available to it. The demand price represents the amount households would be willing to pay for the profits

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[7]One can rationalize implicit contracts without differences in risk preference provided there are limited markets. In an auction market world, workers are unable to diversify their firm specific capital through the use of the labor market. Workers can then diversity their wealth over all industries through the capital markets. Thus there need not be any differences in risk preferences between workers and owners. The similarity of this argument to the discussion of futures markets in the previous section may be noted.

generated by one unit of capital in the project. The present value of a project, and its corresponding demand price will increase if there is an increase in the expected cash flow from the project, a decrease in the effective tax rate on the project's profits or a decrease in interest rates. If investment goods are produced using a technology other than that used to produce consumption goods, or if time and/or resources are required to install new capital, then an increase in the present value of some projects will result in both greater capital accumulation and, at the margin, a price of capital goods equal to the higher demand price of capital. In the steady state equilibrium the demand price of capital will just equal the supply price of capital which is defined as that relative price of capital just necessary to induce the capital goods industry to produce sufficient amounts of capital to maintain the level of capital per worker.

Taxes on capital income, whether collected from the firm or the household, drive a wedge between the marginal product of capital and the effective rate of interest used to make the household consumption decision. Increases in the rate of capital taxation decrease the capital stock, increasing its marginal product. In the dynastic agent case, where the steady state after tax rate of interest is equal to $\delta + n$, this implies that capital owners do not bear any of the burden of the tax. It is also pointed out that shifting the source of a given capital tax rate from households to firms will lower measured interest rates without changing the cost of capital except insofar as households can avoid the capital tax through the increased purchase of durable goods. If such opportunities exist, the cost of productive capital will increase, at least in the short run.

In the face of incomplete markets, the existence of uncertainty implies that agents cannot completely diversify their sources of income. This may suggest that firms, for whom titles to income streams can be traded, will be willing to provide some insurance to their workers through the use of implicit contracts that stabilize real wages, generate layoffs in bad

states and imply that firm profits will be more volatile than the income of other factors of production. It also supports the explanation for the high equity premium based on the concentration of economic shocks on a subset of the population.

There is some indication that investment demand has been strong during the 1980's and may have contributed to the higher than average real rates over that period.[8] Part of the reason for this could be the politically more favorable investment climate in the US as well as the more tangible element of changes in the US tax law that reduced the effective tax rate on corporations and, as well, provided investment tax credits. All these elements worked to increase the expected income from risky assets and, as was noted in Section 4, served to increase the risk free rate of interest as well. Our reading of the data suggests that these factors may have played a role in the period since 1983. We have serious doubts that they were significant prior to that time.

[8]C.F. Blanchard and Summers (1984).

7. GOVERNMENT

7.1 Introduction

In the previous sections of this paper we have discussed the determination of interest rates without a great deal of emphasis on the impact of the government. In this section we look specifically at the role of government in influencing real interest rates. We begin by examining the question of whether or not government debt will alter the steady state levels of interest rates on non–government debt. We argue that there is an impact with all but a few of the modeling assumptions we have used and further that these few are limiting cases that are unlikely to be observed in economies of record. We also spend some time in reconciling the limited impact of government debt on interest rates with our theoretical conclusions.

Governments can also influence the level of interest rates through the type of taxation policy it adopts. We review how the structure of the tax system will influence the behavior of interest rates and, as well, how this might modify prescriptions for appropriate debt policies. We remark also on how government spending and regulatory policies might influence interest rates and the policy implications of these effects.

A second area of inquiry is the normative question of when it might be desirable to engage in a deficit policy to finance a given level of government spending rather than use lump sum taxation. This examines not only the issue of dynamic inefficiency but also the role of government in overcoming the effects of incomplete markets. In this latter case there may be a role for government debt in providing greater diversification of portfolio risk. Because lump sum taxation is often an ideal that is not often available in practice, we also examine the effects of alternative tax policies for given levels of debt to see how this might modify the desirability of debt policy in place of tax policy.

Having specified the mechanisms by which government policies influence, and are influenced by, real interest rates, we survey the influence government may have had on interest rates over the last two decades. We conclude the section with a summary.

7.2 Debt Neutrality

The effect of government debt and deficits on interest rates and capital has been a major source of controversy among North American economists over the last decade, with opinions ranging over the entire spectrum. There are arguments that an increase in government debt will decrease interest rates, that it will increase rates and that it is of no consequence for anything in the economy. Some argue that more debt improves the welfare of future generations; others that it reduces welfare of those generations; and still others, that they could not care less.[1] The majority view that government deficits ultimately reduce the level of private capital relative to what would otherwise prevail is one we share. Nonetheless, it is useful to begin our discussion by examining the case for what is termed the Ricardian Equivalence Theorem (RET), that the method of financing a given level of government spending by bond issue or lump sum taxes is irrelevant to any real economic variable. In particular, the RET states that a decrease in lump sum taxes now that increases debt and debt servicing in the future will not affect the welfare of any agent.

This proposition is most easily shown in the case of certainty with dynastic agents. The representative agent maximizes (3.5) subject to (3.6). Assets held are $a_t = k_t + b_t$ and endowment income is $\omega_t = tr_t + w_t$, where b_t are government bonds outstanding, and paying the same rate of interest as capital, and tr_t are government transfers. For convenience, assume that transfers arise only from operations associated with the issuance of government debt. Their magnitude will be equal to $\dot{b} - (r - n)b$, where $\dot{b} + nb$ are per worker transfers of bonds to the public and rb is the value of lump–sum taxes needed to service the debt. Substituting these values into (3.6) gives:

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[1]For a sampling of the current range of views cf. Barro (1989), Bernheim (1989), and Eisner (1989) in a symposium on budget deficits in *The Journal of Economic Perspectives*. For a narrower focus, the entire September 1987 issue the *Journal of Monetary Economics* is devoted to the effects of government deficits.

$$7.1) \qquad \dot{a}_t = \dot{k}_t + \dot{b}_t = tr_t + w_t + (r_t - n)(k_t + b_t) - c_t$$
$$= (\dot{b}_t - (r - n)b_t) + w_t + (r_t - n)(k_t + b_t) - c_t.$$

There are two methods to show that changes in government debt will not affect agents' choices and therefore will not alter any real economic variable. The first argues that the government is a (transparent) veil and that agents recognize the value and source of their transfers. If there are no redistributions across agents, then (7.1) can be reduced to $\dot{k}_t = w_t + (r_t - n)k_t - c_t$, which is identical to (3.6). Because households can make this calculation, they also recognize that the introduction of government debt will not change their choice set. Because the choice set is unaffected, the first order conditions remain unchanged and thus real variables are unaffected by the existence of government debt and deficits.

The second way to see that debt is neutral in dynastic agent models is to note that an increase in debt of Δb_t increases tangible asset holdings by that amount and simultaneously decreases intangible wealth by the present value of future taxes. But this present value is simply equal to the value of the new government debt, Δb_t. Therefore total wealth is unchanged and, since the budget constraint is unchanged, households will make the same decisions they would have made without the new government debt.

The introduction of uncertainty also does not necessarily lead to the rejection of the RET and debt neutrality. To see this suppose that we take a model that, in the certainty case, has no distribution effects and introduce uncertainty. Then, knowing that there are to be government policies that may provide a random transfer scheme financed by debt and future taxes, agents would collectively form an insurance firm designed to pay all debt servicing taxes. Capitalization of the insurance plan would consist of any random government transfers financed by debt made to members of the plan. Because all agents would see it in their own interest to reduce diversifiable risk, all households would join the plan. Thus, all transfers and all debt servicing would be done between the insurance company and government.

Government debt policy, indeed government itself, would be truly irrelevant![2]

The above demonstration of debt neutrality rests heavily on the absence of distribution effects: the issuance of a unit of debt to a specific decision unit is later serviced by precisely the same unit. As we indicated earlier, dynastic agent models basically assume distribution effects away. If, in fact, there are these effects, then the theorem collapses and government debt is not neutral. This is most easily seen in OG models where these effects play a major role: Suppose that there are b government bonds outstanding and the young are taxed the amount $(r_t - n)b$ to service this debt. Then the capital equilibrium function $r_t = \Psi(w_t)$ given by (3.22) would become

$$r_t = f'(s(w_t - (r_{t-1} - n)b, r_t)/(1 + n) - b)$$

In Figure 3.4 the increase in debt per worker rotates Ψ counter clockwise about $f'(k) = n$, due to the decrease in the savings rate, and shifts it up due to the decreased amount of total saving allocated to capital. Both these effects decrease the steady state capital stock in the normal case of $r_t > n$. Thus the issuance of debt tends, over time, to increase the rate of interest and decrease both the capital stock and the economic welfare of future generations.

It should be stressed that debt neutrality does not rest on the existence of infinite lived decision units. If a system of primogenture is in place then, in a growing economy, new, infinite lived decision units are continually added to the economy and debt neutrality would not hold. Alternatively, if the world was populated by selfish finite lived agents and government debt was rolled over in such a manner that an agent receiving a transfer when young was taxed to pay back principle and interest when old, then debt neutrality would also hold. The RET would also be valid in an OG model if the sale of government debt to the young was used to purchase capital from the old, and the government used the return on capital to service the outstanding debt. In this case, too, no distribution effects occur because

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[2]See Bernheim and Bagwell (1988) on some of the other implications of the assumptions necessary for the RET. It turns out that debt neutrality is among the more innocuous. They argue that, taken as a whole, the implications that are jointly derived are sufficiently out of line with reality that the necessary assumptions cannot hold.

the policy is simply one of introducing a costless to operate financial intermediary into the system.[3]

Establishing the empirical existence of such distribution effects is not, however, a simple matter. One method that has been used is to examine bequests. If bequests represent a significant link between generations then the dynastic agent paradigm with its absence of intergenerational distribution effects may represent an adequate approximation to reality. However, evidence on bequests at death is insufficient to establish the validity of the RET. First of all the theorem does not require that bequests be large, just that they be operative. Further, the transfers need not occur at death. On the other hand, for debt neutrality, the motive for bequests is of some importance. If bequests are accidental, occurring from an unanticipated early death, rather than because of intergenerational altruism, increasing government debt would not induce the increased saving necessary to validate the proposition.[4] Furthermore, Bernheim, Schleifer and Summers (1985) make a strong case for a strategic bequest motive that also vitiates the RET.

Uncertainty combined with incomplete markets may increase the possibility that debt policy will not be neutral. Three cases can be considered. Suppose first that agents cannot insure against future tax levies, and both transfers and debt servicing taxes are distributed randomly across agents with no stochastic correlation between taxes and transfers. This is equivalent to introducing a new asset into the economy with zero expected return and positive risk. An increase in government debt is the equivalent of increasing the mean preserving spread on this security. Our discussion in Chapter 4 indicated that such a change would

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[3]While it is the case that distribution effects vitiate the RET, these are intertemporal distribution effects. A given transfer and tax policy redistributing wealth across agents having different tastes within a generation would have the same impact as an identical transfer policy financed by debt provided the taxes to service and repay the debt had the same intragenerational tax incidence.

[4]In Blanchard's (1985) model, an increase in debt will decrease the capital stock because transfers are capitalized at a higher rate, $r + p$, than is government debt. Thus the increased debt increases wealth and, from (3.17) and (3.20) adjusted for government debt, planned consumption increases at any level of k, decreasing the equilibrium capital stock. This is to say that if $p > 0$, then there is some amount of debt servicing that is paid by decision units not yet alive and this distribution effect is permanent.

increase the demand for assets whose return was uncorrelated with the asset whose mean preserving spread increased. Thus, the increase in debt would reduce the risk free rate of interest as well as rates of interest on some (most?) risky capital goods. The increased debt *decreases* interest rates and increases the capital stock over time![5]

On the other hand, suppose that debt servicing taxes were positively correlated with income; individuals receiving relatively low incomes are taxed less than those receiving high incomes in a period. Then the inclusion of government debt and debt servicing in the portfolio serves as a mechanism to avoid some of the individual specific risk, increasing the risk free rate of interest.[6]

The final case, where transfers are distributed stochastically but debt service charges are perfectly correlated with the transfers provides for the possibility of debt neutrality. Two outcomes are possible. If agents are not constrained in their ability to borrow on future income, then the RET holds: even though there is aggregate uncertainty, once a transfer is received the recipient knows with certainty the share of debt servicing taxes he will face, and the debt he receives in the transfer serves as a perfect hedge for those taxes. The second possible outcome is that the agent receiving the transfer is liquidity constrained as in section 5.4. Then the issuance of government debt serves as a mechanism to relax the constraint (5.7) by essentially converting some of his long term debt into one period marketable debt. This would on average increase the one period rate of interest but at the same time decrease the liquidity premium on two period securities.[7]

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[5]See Chan (1983).

[6]See Barsky, Mankiw and Zeldes (1986).

[7]Some care has to be used here. Yotsuzuka (1987) points out that if the reason for borrowing constraints is asymmetric information on defaults and there are financial intermediaries that issue pooled loans and ration loan size, then the "loans" made by government issued debt are neutral because FI's decrease their loans dollar for dollar. However, Hayford (1989) points out that the empirical relevance of this proposition is limited because bankruptcy generally involves no payments to most creditors.

With the introduction of uncertainty, then, the point remains: If there is no possibility of distribution effects *ex ante* then debt policy will be neutral; if there are possible distribution effects, it will not. These distribution effects arise if there are future generations of taxpayers who are not represented by altruistically motivated parents who receive the initial transfers, or if the servicing costs cannot be allocated in advance by the government, or indirectly by an insurance scheme, to the recipient of the initial transfer.

The second principal reason why government debt policy will influence real rates of interest, and the economy generally, is that taxes are not generally lump sum. We have already discussed the effect of capital taxation in section 6: increases in this tax rate decrease the steady state capital stock and increase the real rate of interest.[8] Thus an unanticipated decrease in taxes now and an increase in taxes in the future to service the debt will increase the real rate of interest in the steady state. If the increase in debt is anticipated then, as was discussed in section 3.5, the impact of the deficit could give rise to a decrease in one period rates prior to the effects of debt servicing. Long term rates would however increase and the capital stock would decline beginning at the time the deficit was anticipated.

In addition, the timing of taxes connected with a deficit may affect the supply of factors of production other than capital. If the labor supply is endogenous, taxes on wage income will give rise to substitution effects. Thus decreasing taxes now and raising them in the future will increase labor supply now and decrease it in the future. Given the transitory increase in labor, the capital–labor ratio will fall and the demand for assets rise, perhaps decreasing the rate of interest at the time of the budget deficit.[9] Thus, the financing decision affects the economy and debt policy will not be neutral.

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[8]This supposes that households cannot completely pierce the government "veil". Transfers associated with any given tax and debt policy will continue to leave (3.6) as the true constraint facing the agent. Thus, if agents take into account the effect on transfers of changing their holdings of capital, the decision problem will continue to be that given by (3.5)–(3.9) and tax, as well as debt, policy will be irrelevant to economic agents.

[9]Bernheim (1989) quotes an experiment by Auerbach and Kotlikoff that decreases income tax rates by a third for 5 years. The result is that savings increase 20% in the first year but the per capita capital stock falls by 7.8% in the steady state.

Finally, as Barro (1974) pointed out in his seminal paper, if there are costs to servicing the debt beyond that of the simple interest payments, the present value of the tax liability will be greater than the value of the debt. This decrease in total wealth may induce agents to consume less now. Given the level of income, this implies a lower rate of interest to induce firms to invest more.

The empirical work on the effect of debt on the rate of interest has centered on the question of the impact of a change in debt rather than the effect of the level of debt per capita. That is, it has looked at the effect of deficits rather than debt per se. If the RET is valid then deficits should not affect interest rates. But our discussion above indicates that even if the level of debt matters, the impact effect may well be ambiguous. As a result it should come as no surprise that the effect of deficits on interest rates suggest that the effect is small.[10] It is even less surprising when it is also noted that measurement of the appropriate level of the deficit has been extremely crude.[11] Public discussion and some empirical work look at the nominal deficit (or the real value of the nominal deficit) without adjusting for the change in the real value of the outstanding debt. Changes in the market value of existing debt due to interest rate changes can be substantial but is often ignored.[12] The value of government assets is also poorly measured, even ignoring the conceptual problems of separating public consumption goods from capital goods; and evaluating implicit commitments to future spending and taxation is often completely ignored. Finally, significant changes in government deficits cannot be viewed in isolation. Generally they have been associated with major economic contractions or with wars. The simultaneity problems associated with deficits thus makes the entire empirical exercise problematic. One is therefore left with how reasonable one believes the assumptions necessary for debt neutrality are. Based on what we regard as

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[10]See Seater (1985), Hayashi (1985), Barro (1987), Boothe and Reid (1989), Evans (1988), and Poterba and Summers (1987) for a sampling of this evidence.

[11]Eisner (1989), and Eisner and Pieper (1984) go into these measurement problems in detail.

[12]See Poitras (1989) for estimates of the market value of government debt in Canada.

reasonable assumptions about actual economies we are reasonably confident that, ceteris paribus, increasing per capita government debt ultimately decreases the capital stock and welfare of future generations. What we do have serious doubts about is whether or not there has been any substantial increase in world government net real indebtedness as a percent of output over the last two decades.

7.3 Tax and Other Government Policies

It is not only debt policies that can affect interest rates. We indicated in section 6 some of the effects of taxation: increases in the tax rate on capital income if paid by households will increase observed interest rates and the cost of capital; increased capital taxes paid by firms, such as the corporate tax, will increase the cost of capital but might not increase observed rates of interest. Barsky, Mankiw and Zeldes (1986) note an additional mechanism by which tax policy can affect interest rates if markets are incomplete. Suppose that all agents are *ex ante* identical but they are unable to completely diversify their portfolio of assets including human capital. Then, levying an income tax and transferring the proceeds equally among all taxpayers will provide a method of insuring individual specific risk. This, in turn, will reduce the observed risk premium for the economy by increasing the risk free rate of interest; it will increase or decrease the cost of capital depending upon the magnitude of agent's CRRA as discussed in section 4. Further, to the extent that it reduces the number of states where agents might otherwise be liquidity constrained it would decrease the implicit rate of interest on illiquid securities and increase it on easily marketable ones in line with our analysis in section 5.

A second type of tax–transfer policy that will affect interest rates is an unfunded social security scheme. To the extent that distribution effects matter this will be qualitatively the same as issuing debt to the elderly and servicing it with taxes on the young. However, if markets are incomplete, there will be some differences. Consider, for instance, the 3 period

OG model developed in section 5. An old age security system that transfers resources from the young and the middle aged to the old would exacerbate the problems of incomplete markets since, in that case, it is the middle aged who bear the costs of the market imperfection and the young who provide some relief to them by purchasing their loans. If these agents cannot borrow on their future social security payment then the middle aged will be more constrained, not less so. In this situation, the risk free rate of interest on liquid assets will be lower with a social security scheme than with an equivalent transfer policy using debt. The cost of capital may however be greater because of the increased liquidity premium.

The regulatory environment facing financial intermediaries will also affect interest rates. Presumably, one of the objectives of FI's is to profit from the establishment of markets that facilitate intertemporal exchange. That these markets are incomplete indicates that the expected revenues from establishing additional markets is less than the expected costs. One source of these costs is government regulation. To the extent that these regulations impede the establishment of additional intertemporal trades, they decrease the rate of interest on marketable securities and increase the implicit return on non–marketable or costly to market ones. The removal of these government regulations would increase the rate of interest on marketable securities.

Because we have concentrated on the determinants of the rate of interest in the long run we have had little to say about the effects of monetary policy on interest rates. We have, however, argued in Section 5 that inflation, or monetary, policy can affect on the relative yields of liquid and illiquid securities: the greater the inflation the greater the spread between these two types of securities. A second effect that deserves mention is the non–indexation of capital income. Under the tax regimes of virtually all countries that tax capital income, the tax applies to nominal capital income. An increase in inflation therefore increases the effective tax rate on capital. This is most easily seen in the steady state of the dynastic agent model. In this case the after–tax real rate will equal $\delta + n$. Thus

$$\delta + n = (r + \pi)(1 - \tau) - \pi = (1 - \tau)r + \tau\pi,$$

or $$f'(k) = r = (\delta + n + \tau\pi)/(1 - \tau)$$

where π is the rate of inflation and will equal the rate of monetary growth less the growth rate of output.

A third way that monetary policy can influence measured real interest rates over extended periods of time is in generating increased uncertainty if markets are not complete. The way to see this is to recognize that inflation is like a tax and, to the extent agents cannot insure themselves against the randomness of the tax and inflation is negatively correlated with output, the larger will be the "equity risk premium." Furthermore the fact that, in a monetary economy, contracts, and most especially debt contracts, are denominated in nominal terms makes it difficult to sort out what the risk free rate might be in a monetarily chaotic world let alone what expected real rates on nominal contracts are.[13]

Finally, in the short run monetary policy appears to exert a very strong influence on both nominal and real rates of interest. The apparent change in the structure of the U.S. Federal Reserves operating procedures in late 1979 may well have been the primary cause of the apparently high real rates of interest in the beginning of this decade.[14]

7.4 Debt Policy

Suppose, as we have argued, government debt policies affect the real economy. Are there any cases in which increases in the average level of debt per capita is desirable? From our discussion in previous parts of this section there appear to be two situations where some intervention may be warranted: One such case is when the economy is dynamically

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[13]There are two points to be made here. First, it strains credibility to believe that all agents should have the same set of inflationary expectations so that, at best, the economy would be characterized by a distribution of beliefs about the expected real return on any given asset. Furthermore, even if agents had shared beliefs on the rate of inflation, to calculate the real rate of return on any asset would require the outside observer to know the entire subjective distribution of inflation in addition to simply the first moment of that distribution. This follows from Jensen's inequality, that $E(1/\tilde{X}) > 1/E(\tilde{X})$. For more on this, see Kochin (1981) or Benninga and Protopapadakis (1983).

[14]Blanchard and Summers (1984) place a heavy emphasis on monetary and fiscal policies as an explanation for the high real rates of interest in the early 1980's.

inefficient. The second may occur if markets are incomplete and debt policy can be used to overcome some of the resulting restrictions on trade. We consider dynamic inefficiency first.

In section 2 we first pointed out the problem of "too much" capital accumulation: If the rate of interest is below the rate of output growth in the steady state, then welfare could be increased for all future generations by decreasing the capital stock: the loss in output would be r, the gain in not having to maintain the per worker capital stock would be $n > r$. Thus some agents would gain and no one would necessarily lose if capital had not been accumulated beyond the point where $r = n$. Thus, if the issuance of government debt can affect the capital stock, increased government debt should be employed whenever $r < n$ so as to maintain $r = n$. Debt issue is not Pareto improving if $r > n$.

The rule, increase debt if $r < n$, may seem a simple one but has generated considerable controversy because, at any point in time, some interest rates are greater, and others less than n. We have argued in section 4.6 that the appropriate rate of interest is the cost of capital and not the risk free rate. This has also been argued by Abel, Mankiw, Summers and Zeckhauser (1989) (hereafter AMSZ). They provide a stringent condition for dynamic efficiency and inefficiency: An economy is efficient (inefficient) if, in all states of nature, the cash flow from capital exceeds (falls short of) the level of gross investment in capital.[15] The intuition behind this condition is quite straightforward: An economy is dynamically efficient if capital supplies more resources for consumption and government spending than it absorbs from the economy under any and all circumstances. Thus a decrease in the capital stock must make some agents sufficiently worse off that they could not, in principle, be compensated for their loss from the gains of others. The dynamically inefficient economy is one where, by reducing the capital

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[15]In a certain world this corresponds to the statement that an economy is efficient (inefficient) if the profits to capital, $(r + \Delta)k_t$, are greater (less) than the gross investment in capital, $(n + \Delta)k + \dot{k}$ where Δ is the depreciation rate on capital. That is, the economy is dynamically efficient (inefficient) if $r_t > (<)\ n + \dot{k}/k$ which is more stringent a condition for dynamic efficiency than $r > n$ when $\dot{k} > 0$, as it has been over the last decade.

stock, output for consumption and government purposes would increase in all future periods. The criterion for efficiency is stringent in that no judgment can be made if there are some states, no matter how remote, where capital absorbs more resources than it generates, then the economy is not necessarily considered efficient since consumers in that state do not, in the aggregate, gain from the existing capital stock.

Despite the stringency of the test for efficiency and inefficiency and the impossibility of determining outcomes in *all* conceivable states of nature, the use of this relatively simple measurement method does provide some suggestive results on the efficiency of the OECD economies for realized states of nature in recent times. It turns out that calculations by AMSZ for the group of seven economies since 1960 indicate that capital's contribution has been everywhere positive. For Canada, over the period 1960–1984, the net contribution of capital has averaged over 10% of GNP and ranged from 7.5% (in 1975) to 17.3% (in 1984). For the seven economies, the lowest contribution was in England in 1974 at 4.4% and, the lowest average (England) was still greater than 9%.[16] Thus we can rule out dynamic inefficiency as a stylized fact for any of the developed economies. We cannot rule out dynamic efficiency. It thus seems reasonable to suggest that, for all practical purposes, decreases in the capital stock in the immediate future would be welfare reducing and suggests, too, that the purposeful crowding out of capital with government debt would not be a desirable policy.

A counter–argument that has occasionally been made in favor of increased debt is that, because the rate of interest on government debt—the risk free real rate—is below the rate of growth of the economy, an increase in that rate would be welfare improving according to the efficiency criteria that $r_i > n$ for all i. There are a number of responses that can be made to this argument. First of all, to the extent that markets are incomplete, the reason for that low rate may reflect a liquidity premium: the total return is the real rate paid on the government securities plus the non–observable return that arises from reducing the costs of exchange. This

[16]U.S. data gathered by AMSZ back to 1929 suggest that capital's contribution was, if anything, greater prior to 1960.

total return could easily be greater than the economy's growth rate.[17] Second, even supposing markets are complete, we would argue that the risk free rate will be an inappropriate guide to the debt financing question. It is inappropriate because it does not take into account the true opportunity cost of an increase in government debt which is the return on capital that is crowded out. To see this note that, by the Modigliani–Miller theorem, the financing decision is irrelevant for the firm. Therefore suppose that firms are financed entirely by equity and the cost of capital is $\hat{R}^m - 1$ prior to the new issue of government debt. The equilibrium condition is given by (4.4): $\hat{R}^m_t = R^f_t(1 + \hat{z}_t)$ where $\hat{z}_t$ is the risk premium. To induce agents to hold the new government debt, R^f_t must rise, increasing $\hat{R}^m_t$ by the factor $(1 + \hat{z}_t)$. This decreases the demand price of capital and reduces investment below what it otherwise would be. Thus $\hat{R}^m$ is the opportunity cost of increasing government debt. If $\hat{R}^m$ is greater than one plus the growth rate of the economy increased debt will reduce welfare.

To say that the economy is dynamically efficient is not to say there is no role for government debt policy in the long run. If there are incomplete markets such that agents cannot diversify completely, or transactions costs that can be reduced by providing liquid assets, then a role for debt might still be possible. In the case of transactions costs in a certain world, the optimal level of debt would be where the marginal resource cost of maintaining an additional real unit of government debt equals $r - r_b$. This cost includes not only the cost of operating the bond desk and related expenses, but also includes any distortions and collection costs that might arise from servicing the debt. Furthermore, given that FI's represent an alternative mechanism by which these "monetary services" can be provided, it is not at all clear that there remains much role for debt policy. More practical perhaps would be structuring the regulatory framework for FI's to provide those institutions with the ability to provide these services competitively and efficiently.

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[17]Cf. section 5.3 above, or Fried and Howitt (1983).

Similar statements apply to the use of debt policy to permit greater diversification. Financial intermediaries also have an incentive to improve the operations of markets to the point where the marginal cost of any additional improvement no longer covers the price agents are willing to pay for the additional services. It may well be that the government has a comparative advantage in coping with some of the externalities in establishing more efficient markets but, until these are identified, the government should be sparing in its use of debt for this purpose. Again there may be better methods of providing diversification: The use of income or consumption taxes may be one such method.

7.5 Summary and Conclusions

In this section we have examined the effects of government policies on the level of interest rates. One area of special interest is debt policy and the possibility of government debt crowding out private capital formation by increasing the cost of capital. We argued that an increase in government debt as a percent of GNP will ultimately increase interest rates and decrease the capital stock relative to what it otherwise would be if the same level of government spending had been financed with taxes instead of debt issue. To have no effect on the capital stock requires an environment where there are no intergenerational redistribution effects, no distorting taxes, complete markets and government financial intermediation be a perfect substitute for private financial intermediation. Our view of the economy is that these conditions simply do not exist: Generations are not so benevolently linked that families fully undo the intertemporal redistributions implicit in greater debt issue and, as a consequence, capital accumulation is lower than it otherwise would be. Tax systems are generally income linked so that the increased tax rate on capital needed to service the debt will drive up the before tax rate of interest. If markets are not complete the increased income tax rate will decrease individual specific risk increasing the risk free rate of interest. Finally, because of the power of taxation it is unlikely that government intermediation services and those provided by private financial intermediaries are identical.

Despite the strong theoretical presumption that increased debt ultimately increases observable rates of interest, empirically, the evidence available is quite mixed. One reason why this is the case is that because changes in real debt per capita are generally small, economists have instead looked at the issue of the effects of deficits rather than debt. Doing so, however, is full of pitfalls. First of all, economists rarely have the opportunity to run controlled experiments. Major changes in the deficit are usually associated with major historical events such as wars and major economic contractions. Disentangling the effects of these events from thos of the deficit is difficult in the extreme. Secondly, anticipations of future deficits can also obscure the relationship between a current deficit and interest rates. Third, measurement of the change in the real net worth of the government which is the theoretical construct applicable to the theory bears little correlation to the nominal deficit. Only recently have the appropriate adjustments been made and the empirical effects depend critically on which adjustments have been made to the data.

We also indicated that tax policies played a role in determining interest rates: In addition to the effects of tax policy noted in section 6, tax policy can provide a means for agents to increase diversification if private markets are not complete. For a given level of revenue raised, the risk free rate of interest will be greater the less random across agents taxes are perceived to be and the greater the correlation of tax revenue with the individual's level of income and/or consumption.

Regulatory policies on financial institutions will also have an effect on rates in this environment. The more competitive these institutions, the greater their ability to diversify their portfolios, the more they are able to reduce the costs of operating in financial markets, and the better able they are to compete for funds, the lower the wedge between the cost of capital to a representative firm and representative household will be. The development of the high yield bond market in the U.S. represents a reduction in the cost of funds to firms operating in that market relative to their alternative sources of funds prior to establishment of

this market. The development of financial futures markets provide agents further methods to diversify and thereby reduce the cost of funds to larger firms; restrictions on portfolio allocation of some intermediaries to regional firms or to certain asset types, reduce the ability of households to diversify risk as completely as they might wish through their FI's.

Monetary policy can influence real interest rates first by indirectly altering the marginal flow of monetary service available from near monies: The greater the rate of monetary growth, the greater the liquidity yield and the lower the observed real rates of interest on these near–monies. Second, in the absence of inflation indexing of capital gains and losses, increases in inflation will ultimately increase the real before tax return on assets. Third, to the extent that monetary policy cannot be anticipated, the risk premium on securities paying fixed nominal interest and principal will increase relative to those assets less subject to the uncertainty attached to changes in the price level. Finally, the short–run effects of unanticipated changes in the stock of money have the traditional effects on short–term rates of interest that, while important in describing interest rate behavior over the cycle, has little effect in the longer term.

The last topic we addressed in section 7 was the normative question of when it might be beneficial to increase government debt. We argued that, in principle, debt issue would be desirable if the cost of capital was less than the rate of output growth, such a situation has not existed in any major economy for the last 30 years and has not likely existed at any time in this century. Furthermore, it was stressed that the opportunity cost of government debt is the cost of capital and not the risk free borrowing rate for the government. Finally, while there may be a role for government debt in overcoming the problems of incomplete markets, the establishment of either a stable income or consumption based tax system or the encouragement of an efficient, competitive financial industry could prove to be a more efficient method of dealing with these problems.

These results suggest that government actions had significant effects on interest rates over the last two decades. The low real rates on short term securities in the 1970's can easily be attributed to the high level of inflation over that period; the historically high rates since 1982 can be traced, at least in part, to the reduction in the inflation rate and to the move to financial deregulation. Both of these sources of higher rates, we would add, can be viewed positively for economic welfare as both appear to reduce the economic costs of incomplete markets. On a less positive note, the cumulative increase in government debt may have marginally increased interest rates since 1981 at the expense of some capital formation. Given that the world economy is not now nor expected to be dynamically inefficient any time soon such debt policies are unlikely to be welfare improving for future generations. Finally, the rise in real interest rates from 1980 until some time in 1982 appears to have been the result of the change in monetary policy over that time period. This shift can be viewed as causing a transitory increase in real rates of interest and appears to be one of the crosses market economies must bear given the current structure of information and exchange.

8 THE OPEN ECONOMY

8.1 Introduction

The previous sections of this essay have examined the determinants of the real rate of interest in a closed economy. This section considers how the real rate of interest is determined in an open economy. We begin by focusing on the case of a small open economy that can import and export all tradeable goods at given world prices, and whose international borrowing and lending has no influence on the "world" real interest rate. Under conditions of perfect foresight the domestic real interest rate will be independent of both the thriftiness of domestic wealth–holders and the productivity of the domestic technology. Rather, it will be determined by productivity and thrift in the rest of the world and by the tax treatment of capital income. We examine the implications for government debt policy of the small open economy's growth rate being greater than, equal to or less than the real interest rate.

We then relax the assumption that the country cannot influence commodity prices and suppose instead that it has market power in its export market. In this "almost small" open economy changes in government debt policy will affect the real exchange rate and, in the transition from one equilibrium exchange rate to another, permit the domestic rate of interest to diverge from the world rate by the expected change in the real exchange rate. Assuming that the economy is dynamically efficient, the steady state burden of government debt will be higher than in the small open economy because of the permanent effect of government debt on the real exchange rate.

8.2 Basics

Suppose that the economy produces a single, composite tradeable good that can either be consumed, invested or exported to service foreign loans. The domestic labour force grows at rate n, and the prevailing world real interest rate is r^* which is assumed to remain constant over time. In the absence of capital income taxes at home or abroad, and under conditions of

perfect foresight, the world real interest rate will reflect both the marginal rate of time preference and the marginal productivity of capital in the rest of the world. The domestic capital stock will adjust so that its marginal rate of productivity is always equal to the world real interest rate, and domestic wealth holders will make their intertemporal consumption–savings plans so that their marginal rate of time preference always coincides with the world real interest rate. The domestic real interest rate (which reflects both the domestic marginal rate of productivity of capital and the domestic marginal rate of time preference) will then always coincide with the world real interest rate.

If we write the technology in intensive form we have the condition $f_t'(k_t) = r^*$ for all t, which fully determines the capital intensity of domestic production. If we define $R_t^* = 1+r_t^*$, $\sigma = -u'(c_t)/c_t u''(c_t)$, (where σ is the elasticity of the marginal utility of consumption at each date) and $\beta = (1 + \delta)^{-1}$ (where δ is the "pure" rate of time preference of domestic wealth–holders, or alternatively the coefficient of relative risk aversion) we have the condition $\beta R_t^* = (c_t / c_{t+1})^{-\sigma}$, which fully determines the ratio of present to future consumption. Since changes in β or σ do not affect k_t, the thriftiness of domestic wealth–holders has no influence on the level of the domestic capital stock or the domestic real interest rate. On the other hand, changes in $f_t'(\cdot)$ (arising from technological innovations or changes in the terms of trade) will affect k_t and also affect c_t and c_{t+1}, and therefore savings in period t. But the impact of changes in $f_t'(\cdot)$ on the intertemporal consumption stream occurs only through wealth effects; there is no change in the ratio c_t / c_{t+1} because there is no change in the world and domestic real interest rate.

8.3 The Determinants of International Indebtedness

While the thriftiness of domestic wealth–holders is irrelevant in determining the domestic real interest rate, it is crucial in determining whether the economy is a net creditor or debtor in the steady state. Consider the diverse implications of the infinitely lived versus overlapping generations paradigms.

The infinitely lived representative family grows at rate n and maximizes (3.5), $\int_0^\infty u(c)e^{-\delta t}dt$, subject to $\dot{a} = (r^* - n)\,a + w_0 - c$, where c is consumption per person, a is assets per person, and $w_0 = f(k_0) - k_0 f'(k_0)$ is the real wage, which is constant through time since $f'(k_0) = r^*$. The optimal consumption plan satisfies $\dot{c}/c = \sigma(r^* - n - \delta)$, which says that consumption per person increases, remains constant or decreases over time depending upon whether $r^* - n \gtrless \delta$. If $r^* - n > \delta$ the "small" open economy evolves from a net debtor to a net creditor and it eventually owns the world. Conversely, if $r^* - n < \delta$ consumption per person in the small open economy goes to zero in the long run. Thus, there is no steady state equilibrium for the small open economy unless $\delta = \delta^*$, i.e. unless the intertemporal preferences of individuals in the small open economy exhibit the same degree of impatience as preferences in the rest of the world.[1]

It is also clear that a steady state equilibrium for the small open economy with a positive pure rate of time preference is not possible unless the world real interest rate exceeds the economy's rate of growth of real output. Even if there is no steady state, $r^* > n$ is necessary for the small open economy's initial wealth to be finite so as to avoid boundless consumption. This follows because initial wealth $V_0 = \int_0^\infty N_t w_0 e^{-r^* t}dt + a_0$ where N_t is defined by (2.1). Thus $V_0 = w_0/(r^* - n) + a_0$ which is finite if and only if $r^* > n$.

In section 3 we discussed two ways out of the "knife–edge" situation in which a steady state for the small open economy exists only if its pure rate of time preference coincides with that in the rest of the world: make δ a function of future utility levels, or include asset holdings in the instantaneous preference function. A third way is to make δ depend (positively) on the level of per capita consumption, i.e. $\delta(c)$ where $\delta'(c) > 0$. The optimal consumption plan then satisfies $\dot{c}/c = \sigma(r^* - n - \delta(c))$, and there exists a $\bar{c}$ such that

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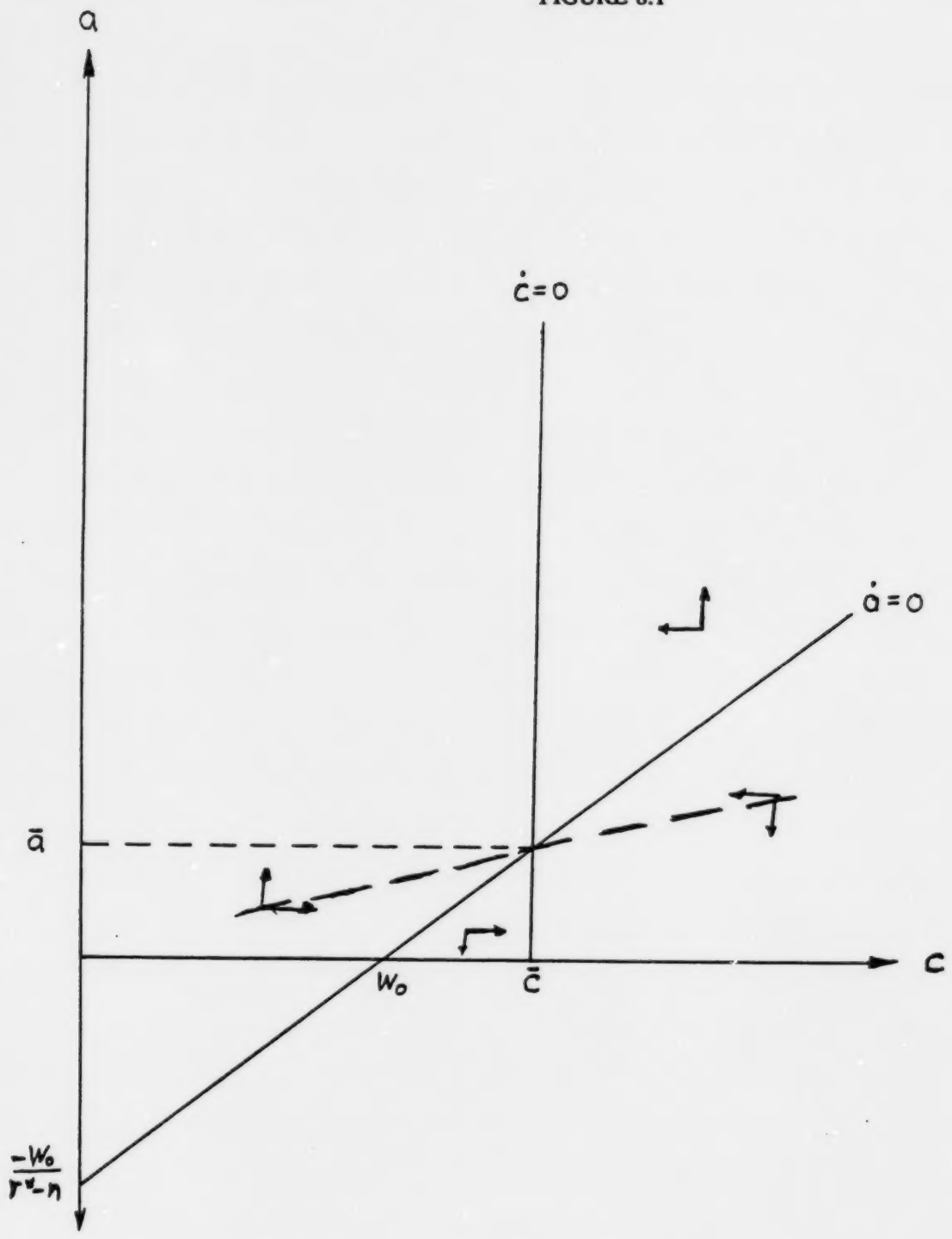
[1]This conclusion depends upon the assumption that the rest of the world is also growing at rate n. Since the rest of the world is essentially a closed economy, the world real interest rate must satisfy $r^* - n = \delta^*$ in the steady state.

$\delta(\bar{c}) = r^* - n$. Thus the small open economy attains a steady state by having its rate of time preference adapt to the rate prevailing in the rest of the world.

Figure 8.1 provides a convenient representation of the unique, saddlepoint stable consumption plan. Since there is only one value of c for which $\delta(c) = r^* - n$, the $\dot{c} = 0$ isocline is vertical at $c = \bar{c}$. The $\dot{a} = 0$ isocline is linear with slope $(r^* - n)^{-1}$ and horizontal intercept w_0. In the steady state $\bar{a} = (\bar{c} - w_0)/(r^* - n)$. If initial non–human wealth per capita a_0 is less than $\bar{a}$, initial consumption per capita c_0 will be less than $\bar{c}$ and consumption and non–human wealth per capita will accumulate. On the other hand, if $a_0 > \bar{a}$, c_0 will exceed $\bar{c}$ and consumption per capita and non–human wealth per capita will both fall. The economy will be a net creditor or debtor in the steady state depending upon whether $\bar{a} \gtrless k_0$, where k_0 satisfies $f'(k_0) = r^*$. Ceteris paribus, the more "impatient" the economy (i.e. the higher is $\delta(c)$ at any level of c) the further to the left will be the $\dot{c} = 0$ isocline. The $\dot{a} = 0$ isocline is independent of domestic thriftiness. Therefore, the more impatient the country the smaller will be its steady state per capita non–human wealth $\bar{a}$. Since k_0 depends only on the productivity of the technology and r^*, the more impatient the country the more likely will it be a net debtor in the steady state.

In the two period overlapping generations paradigm the real income of a member of the old (retired) generation will be determined by the amount of savings undertaken in the previous period at the world real interest rate r^*. The real wage of a member of the young generation will be determined, as before, by the amount of capital per worker that can be profitably invested in the domestic economy at the world real interest rate, i.e. $w_0 = f(k_0) - k_0 f'(k_0)$ where k_0 satisfies the condition $f'(k_0) = r^*$. If the intertemporal preferences of the representative individual are given by $u(c_1, c_2) = c_1^{\gamma} + (1 + \delta)^{-1} c_2^{\gamma}$ and there are no bequests, the individual maximizes $u(\cdot)$ subject to the constraint $c_1 + c_2/(1 + r^*) = w_0$. The first order condition for an interior maximum gives $c_2 = c_1[(1 + r^*)/(1 + \delta)]^{1/(1-\gamma)}$ and it follows that savings per worker (equivalently, assets per worker) will be proportional to the real wage, with the proportionality factor positively or negatively sensitive to the real interest rate depending

FIGURE 8.1



Phase path to steady state in small open economy model

upon whether $\gamma \gtrless 1$; i.e., $a = (w_0 - c_1)/(1+n) = \alpha(r^*)w_0$. As in the infinitely lived agent paradigm, the economy will be a net creditor or debtor in the steady state depending upon whether assets per worker are greater than or less than capital requirements per worker. However, unlike in the infinitely lived agent paradigm, the small open economy will attain a steady state equilibrium no matter what the value of δ, i.e. no matter whether individuals in the small open economy exhibit a higher or a lower degree of intertemporal impatience than individuals in the rest of the world. Moreover, a steady state equilibrium exists whether $r^* \gtrless n$ since the consumption plans of finite lived agents are bounded for any r^*.

8.4 Government Debt, International Indebtedness, and Welfare

If the domestic real interest rate is determined exogenously in the small open economy, it can't be affected by government debt and government deficits. The issue we address in this section is: How do government debt and deficits affect net foreign indebtedness and national economic welfare in the small open economy? In the infinitely lived representative agent model government debt has no effect on net foreign indebtedness since Ricardian equivalence makes debt irrelevant (assuming that agents have perfect foresight). An increase in government debt at time t_0 corresponds to a reduction in (lump–sum) taxes at time t_0, but in order to service the debt taxes will have to be increased in all future periods. The present value of the change in taxes is zero. Since government debt has no real effects on the private sector's consumption plan or private sector capital formation it has no effect on the balance of payments and the steady state burden of government debt is zero.

In the 2 period, overlapping generations model with no bequests government debt will increase net foreign indebtedness and the steady state burden of government debt will be $r^* - n$. The proof proceeds as follows. The after tax wage of a young worker is $w_0 - \tau$, where τ is (lump–sum) taxes per worker, $w_0 = f(k_0) - k_0 f'(k_0)$, and k_0 satisfies $f'(k_0) = r^*$. Steady state assets per worker reflect the savings of the young and must satisfy the condition $(1+n)a = w_0 - \tau - c(w_0 - \tau, r^*)$, where $c(\cdot)$ is consumption per worker. Assuming for

simplicity that the government's role is purely redistributive, the steady state government budget constraint will satisfy the condition $(r^* - n)g - \tau = 0$, where g represents government debt per worker. Thus, the tax on each worker's income is just enough to service the existing level of debt and prevent it from growing faster than the economy. Finally, steady state net foreign indebtedness per worker will be the difference between capital per worker plus debt per worker minus assets per worker so $e = k + g - a$.

If lump sum taxes are cut to the current young generation there will be an increase in government debt that will require an increase in lump–sum taxes on all future generations to prevent government debt per worker from growing further. From the government budget constraint we see that $d\tau/dg = r^* - n$, and since the pre–tax wage is unaffected we have $d(w_0 - \tau)/dg = -(r^* - n)$. From the assets–savings relationship we see that $(1+n)da/dg = -(1-c_w)d\tau/dg = -(1-c_w)(r^* - n)$. Finally, from the capital market equilibrium condition we see that $de/dg = 1 - da/dg = 1 + (1-c_w)(r^* - n)/(1+n)$. Therefore, an increase in government debt reduces the steady state lifetime real income of an individual by $-(r^* - n)$, reduces steady state assets per worker by $da/dg = -(1-c_w)(r^* - n)/(1+n)$, and increases steady state net foreign indebtedness by more than the increase in government debt itself. Even though government debt does not crowd out the domestic capital stock it does reduce the economy's pool of savings because it makes future generations poorer. Therefore, an increase in government debt has a magnified effect on net foreign indebtedness.[2]

In sum, no matter what the micro foundations for the thriftiness of individuals, for a small open economy the real interest rate is determined in the rest of the world. However, the impact of government debt on the balance of payments, and the steady state burden of government debt, depends crucially upon how individuals behave. If individuals act as if they live forever then there will be no impact of government debt on the balance of payments and the steady state burden of government debt will be zero. However, if individuals have finite

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[2]The welfare effects of government debt in a small open economy (and also in a large open economy which has some influence over the world real interest rate) are discussed in Persson (1985).

lives and make no bequests then government debt will increase net foreign indebtedness by a magnified amount and the steady state burden of government debt will be $r^* - n$.

8.5 Effects of Taxation

According to the analysis up to this point the domestic real interest rate will coincide with the world real interest rate no matter what the determinants of the thriftiness of domestic wealth–holders, the productivity of the domestic technology, or the size of government debt and deficits. In this section we consider whether and to what extent the tax treatment of capital income can affect the real interest rate in the small open economy.

Since the capital income tax drives a wedge between the domestic marginal rate of productivity of capital and the domestic marginal rate of time preference we first have to decide what we mean by the domestic real interest rate. Does it reflect the cost of capital to domestic firms, the reward for savings to domestic wealth–holders, or something in between? Typically, the domestic real interest rate will be driven below the cost of capital by the corporate income tax but driven above the return for savings by the personal income tax. In the interest of simplicity we assume full integration of the corporate and personal income tax such that the domestic (host) country imposes a single, *ad valorem* tax at rate τ_k on all capital income generated within its borders. We also continue to follow the convention of defining the domestic real interest rate as the cost of capital to domestic firms. Thus $r_t = f_t'(k_t)$, whereas $(1-\tau_k)r_t = r_{bt}$ is the reward for savings.

The domestic rate of tax on capital income may or may not affect the domestic real interest rate depending upon the details of the international tax code. Suppose that each country (home and foreign) imposes its own rate of tax on capital income (τ_k and τ_k^* respectively), but each country permits all taxes paid to the other country on the capital income of its residents to be fully credited against taxes owed at home. Under these conditions changes in the domestic (home) country tax rate when $\tau_k < \tau_k^*$ will leave the cost of

capital to domestic firms unaffected and thus have no influence on domestic capital formation. This follows because the marginal supply of savings to finance domestic capital formation is foreign and capital in the home country must yield an after tax rate of return to foreign savers of r_b^*, namely the after tax rate of return on capital held by foreigners in their own country. Specifically, the condition $(1-\tau_k^*)f'(k) = r_b^*$ must hold no matter what the value of τ_k (provided that $\tau_k < \tau_k^*$).

On the other hand, changes in the domestic tax rate when $\tau_k < \tau_k^*$ will affect the rate of return on domestic savings. Since $r_b = (1-\tau_k)f'(k)$, and since $f'(k)$ is unaffected by changes in τ_k, it follows that $dr_b/d\tau_k = -f'(k) = -r_b^*/(1-\tau_k^*) < 0$. In a model in which agents have perfect foresight r_b will reflect the *financial* cost to the government of issuing government debt since it is the net of tax yield that domestic wealth–holders will require to willingly hold government debt. Thus we have the important result that a small increase in τ_k when $\tau_k < \tau_k^*$ leaves the cost of capital to domestic firms unaffected but lowers the financial cost to the government of issuing government debt.

Now consider what happens to our results for variations in τ_k when $\tau_k > \tau_k^*$. Assuming that the marginal supply of savings to finance domestic capital formation is still foreign, the effective rate of tax on capital income is now the higher domestic rate τ_k. The domestic capital stock must yield an after tax return to foreigners equal to the return they can earn in their own country, so we have the condition $(1-\tau_k)f'(k) = r_b^*$. If we differentiate this condition with respect to a small change in τ_k we have $df'(k)/d\tau_k = f'(k)/(1-\tau_k) > 0$, which says that an increase in τ_k when $\tau_k \geq \tau_k^*$ will increase the cost of capital to domestic firms and deter domestic capital formation. Insofar as the impact on the rate of return on domestic savings is concerned, if we differentiate the condition $r_b = (1-\tau_k)f'(k)$ with respect to a small change in τ_k we find that $dr_b/d\tau_k = (1-\tau_k)df'(k)/d\tau_k - f'(k) = 0$. Therefore, we conclude that changes in τ_k when $\tau_k > \tau_k^*$ will not affect the rate of return on domestic savings, and therefore will not affect the financial cost to the government of issuing government debt.

Our conclusion up to this point is that changes in the tax rate on capital income in the small open economy with a fully operative foreign tax credit will either change the cost of capital to private firms or the financial cost to the government of issuing government debt, but not both. Two other important results follow. First, if the small open economy wants to choose its rate of tax on capital income so as to maximize national economic welfare it should set $\tau_k = \tau_k^*$ so as to achieve international tax harmonization. If τ_k is set below τ_k^* there is a transfer of tax revenue from the home to the foreign government, whereas if τ_k is set above τ_k^* there is a reduction in the domestic capital stock which is already below its socially optimal level (i.e. the value of k which satisfies $f'(k) = r^*$). Second, insofar as government debt policy is concerned the value of τ_k matters. If $\tau_k \geq \tau_k^*$ the steady state burden of government debt is $r_b^* - n$, the difference between the after tax rate of return demanded by foreign savers and the economy's growth rate of real output. However, if $\tau_k < \tau_k^*$ the steady state burden of government debt is $r_b - n > r_b^* - n$ because government debt absorbs domestic savings and thereby transfers tax revenue from the home to the foreign government.

8.6 Effects of Inflation

In this section we consider the impact of domestic and foreign inflation on the real interest rate in the small open economy.[3] We continue to assume that the home and foreign country impose the same rates of tax on capital income, the corporate and personal income taxes are fully integrated, and that taxes paid in the host country are fully credited against taxes owed in the source country. We also assume that capital gains and losses from exchange rate changes are taxed at the same rate as interest and dividend income and the exchange rate always moves to preserve purchasing power parity. Let i and i^* represent the nominal interest rates at home and abroad, and π and π^* represent the respective inflation rates. The real cost of capital to domestic firms r is the difference between the nominal interest rate and the

[3]The analysis of this section draws on the work of Hansson and Stuart (1986).

inflation rate, i.e., $r = i - \pi$, and the real after tax rate of return on domestic savings r_b is the after tax nominal interest rate minus the inflation rate, i.e., $r_b = i(1-\tau_k)-\pi$. $\pi-\pi^*$ represents the rate of appreciation of the foreign currency vis a vis the domestic and the following equation links home and foreign nominal interest rates in a perfect capital market:

8.1) $i - i^* = \pi - \pi^*$.

Thus, the nominal interest rate differential must equal the (anticipated) exchange rate change. It follows that the nominal interest rate will move one for one with the domestic inflation rate in the small open economy no matter what the common rate of tax on capital income.

Suppose that the domestic and foreign inflation rates are both initially zero, the nominal and real interest rates are each 4 percent (which reflects the real cost of capital to domestic and foreign firms), and the marginal tax rate on capital income is 50 percent so the real after tax rate of return to savers is 2 percent. If the domestic inflation rate increases to 4 percent per annum while the foreign inflation rate remains at zero, the domestic nominal interest rate must increase to 8 percent in order to maintain the real after tax rate of return at 2 percent for foreign savers. (Foreigners earn a nominal return of 8 percent but are allowed to deduct capital losses from exchange rate changes of 4 percent before paying taxes at the 50 percent rate.) As a consequence, domestic inflation has reduced the real after tax rate of return to domestic savers; they earn a nominal return of 8 percent and must pay taxes at the 50 percent rate on all of this income (since they have no capital gains or losses from the exchange rate change) so their real after tax rate of return is zero after deducting the 4 percent inflation rate. The real cost of capital to domestic firms has remained unchanged at 4 percent. The real cost of capital to foreign firms is also unchanged at 4 percent, and the real after tax rate of return on foreign savings is unchanged at 2 percent. Therefore, domestic inflation does not create a real interest rate differential between countries, but it does create a differential between the after tax rates of return on savings at home and abroad.

Now consider the impact of *foreign* inflation on the real interest rate in the small open economy. The rest of the world can be viewed as a closed economy insofar as the determination of the foreign nominal and real interest rates are concerned. For simplicity, assume that foreign savers are willing to supply funding only if they earn a real after tax rate of return of 2 percent so that r_b^* is given. Under these conditions the nominal and real interest rates abroad are related to the foreign inflation rate and the tax structure as follows:

8.2) $$i^*(1-\tau_k^*) - \pi^* = r_b^*$$

If the real after tax rate of return to foreign savers r_b^* is to remain invariant to the foreign inflation rate, then π^* and i^* must move so that

8.3) $$di^* = d\pi^*/(1-\tau_k^*)$$

Assuming that t^* is 50 percent and π^* increases from zero to 4 percent per annum, i^* must increase by 8 percentage points, i.e. from 4 percent to 12 percent. The impact on the foreign real interest rate is then given by $dr^* = di^* - d\pi^*$ and substituting (8.3) we have

8.4) $$dr^* = d\pi^*[\tau_k^*/(1-\tau_k^*)]$$

Assuming that there is no domestic inflation so that π=0 the impact on the domestic nominal interest rate is found by differentiating (8.1) to obtain:

8.5) $$di = di^* - d\pi^*$$

Substituting (8.3) into (8.5) we obtain

8.6) $$di = d\pi^*[\tau_k^*/(1-\tau_k^*)]$$

If $\tau_k^* = .5$ then the domestic nominal interest rate moves one for one with the foreign inflation rate. Since there is no domestic inflation the nominal and real interest rates are identical. Therefore foreign inflation raises the real interest rate at home and abroad by the same amount. The real after tax rate on return on domestic savings r_b also increases as a

consequence of foreign inflation, but if the tax rate is 50 percent r_b will increase by only 2 percentage points when the foreign inflation rate rises from zero to 4 percent. Since the real after tax return on foreign savings is unchanged at 2 percent, foreign inflation also creates a differential between the after tax rates of return on savings at home and abroad.

To summarize, *domestic* inflation in the small open economy leaves the real cost of capital to domestic firms unaffected, but it lowers the real after tax rate of return on domestic savings. On the other hand, *foreign* inflation raises the real cost of capital to domestic firms one for one and raises the real after tax rate of return on domestic savings by a proportion equal to one minus the marginal tax rate on capital income. Thus differences in inflation rates between countries can create differences in real after tax rates of return on savings but not differences between real interest rates.

8.7 Nontradeables

Suppose that some of the output of a small open economy cannot be traded owing to transport costs etc. Many types of services fall into this category since they cannot be stored (except when they are embodied in goods as with films, computer tapes etc.) so production and consumption must occur at the same time, if not the same place. The prices of nontradeables are not set on world markets but are free to vary according to conditions of domestic demand and supply. Suppose that tradeables can either be consumed or invested while nontradeables can only be consumed, and that capital and labour are the only scarce factors in the economy and both are freely mobile between sectors. Then capital will be utilized in the production of tradeables such that $f'(k_T) = r^*$, and capital will be utilized in the production of non–tradeables such that $h'(k_N) = r^*/p$, where p represents the relative price of non–tradeables to tradeables, or the real exchange rate.

Given the Heckscher–Ohlin structure of the model the real wage and rental rate will be fully determined in the tradeables sector and there is only one value of p that makes it

economic to use capital and labour in the production of non–tradeables. Under these conditions the real exchange rate will be unaffected by demand conditions in the non–tradeables sector and the domestic real interest rate will always coincide with the world real interest rate. However, if one factor is immobile between sectors in the short run then the real exchange rate will be affected by any policy that switches spending on national output, such as a cut in lump sum taxes to the current young generation in an OG model, which results in a permanent increase in government debt. If p changes between periods then the domestic real interest rate (which reflects the marginal rate of substitution between current and future consumption) will deviate from the world real interest rate (which reflects the marginal rate of substitution between current and future consumption of tradeables only).[4] Specifically, r will exceed r^* if p increases; and r will fall below r^* if p decreases. However, such divergences are only temporary since in the long run labour will shift between sectors to equilibrate wage rates and thereby restore the real exchange rate to its long run equilibrium value.

If some factors are immobile between sectors in the long run an expenditure switching policy (such as a cut in lump sum taxes to the young which results in a permanent increase in government debt) will cause a permanent change in the real exchange rate, but r will deviate from r^* only during the transition. The steady state burden of an increase in government debt will be the same as in the simple one sector OG model of the small open economy (namely $r^* - n$) as long as resources are efficiently allocated between tradeables and non–tradeables to begin with. Any intersectoral resource re–allocation that takes place as a result of the increase in debt will not, of itself, produce a welfare gain or loss.

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[4]Dornbusch (1983) provides a useful discussion of this point.

8.8 Uncertainty

The analysis up to this point is based upon the assumption that agents have perfect foresight. In such an environment we have found that there are limited opportunities for real interest rates to diverge between countries: taxes, capital controls, and non–traded goods are the most notable. Even these explanations have their problems. Taxes might explain deviations between real interest rates, but once the tax structure is set there is no way to explain variations in the differential over time. The existence of non–traded goods can explain *temporary* divergences between measured real interest rates in response to government deficits, but not persistent divergences. Finally, capital controls provide an explanation for persistent and time varying differential yields, but only by eliminating the international capital market from the picture.

If we drop the assumption of perfect foresight we can explain the existence of persistent and time varying real interest rate differentials between countries even if there are no taxes on capital income, no capital controls, and no nontraded goods. The explanation rests on the notion of "country risk", the idea that foreign lenders perceive a greater risk of default (or rescheduling) on their loans the greater the ratio of net indebtedness to GDP of the borrowing country.[5] However, there are two further requirements for this explanation to work. First, we are talking about loans that are backed by the government of the borrowing country, since otherwise there would be no reason why an individual lender would worry about the borrowing country's "macro" position in assessing the riskiness of his or her loan to an individual borrower. Second, either the risk perceptions of foreign lenders must not be matched by the perceptions of domestic borrowers—an asymmetric perception of default (or rescheduling)—or there must be systematic differences in the portfolios of domestic residents and foreigners. If there are systematic portfolio differences it implies, as in section 6, that markets are incomplete and residents and non–residents cannot diversity over all assets.

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[5]The micro foundations for the country risk premium are discussed in Harberger (1980) and Eaton and Gersovitz (1981).

There is good reason to believe that such market failures occur in a world with many independent (and democratic) countries. In particular, governments can only tax their own residents (or to a limited extent, those earning income in the host country). These tax liabilities are a part of the portfolio of the residents of the economy and are necessarily held in different proportions by nationals of different states. It further means that the net wealth position in a country's government debt is less for residents of the country than for others.[6] Because nationals cannot sell their votes to foreigners and governments cannot commit their taxing power as collateral, a moral hazard problem arises that would generate the country risk premium. Further, the greater the level of foreign indebtedness the greater the likelihood of a potential default.

The existence of country risk premia can explain why debtor nations have higher real interest rates than creditor nations even in a perfectly functioning international capital market. We are then back to the position that an individual country's real interest rate depends upon the thriftiness of its citizens and the productivity of its technology after all: the greater a country's thriftiness relative to the productivity of its technology the less it will rely upon external funding to finance its capital requirements, the lower will be its net indebtedness to GDP ratio, and the lower will be the premium it will have to pay above the world real interest rate to attract foreign loans.

8.9 The Almost Small Open Economy

A country may be able to borrow and lend at a given world real interest rate (plus, perhaps, an exogenous country risk premium) and purchase imports at given world prices, but it may also face downward sloping demand schedules for its exports because they are perceived to be imperfect substitutes for the tradeables of other countries. Furthermore, the

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[6]Suppose θ and $1 - \theta$ are the proportions of government debt of country A held by residents and non-residents respectively, $0 < \theta < 1$, and that taxes are levied only on a country's residents. Then residents net position in the debt of A is $\theta - 1 < 0 < 1 - \theta$.

export industries may consist of several firms and therefore be approximately competitive so whatever market power the country as a whole possesses will not be fully exercised by the firms themselves; it will require some form of government intervention (e.g., export taxes or quotas).

In this "almost small" open economy an increase in government debt causes a permanent deterioration in the terms of trade (real exchange rate) and as a result a permanent reduction in the pre–tax real wage. The steady state burden of government debt is therefore larger than in the small open economy. Also, government debt may either crowd out or crowd in domestic capital formation in the long run so there are additional welfare effects of government debt when there are taxes on capital income. Finally, insofar as transitional effects are concerned, an increase in government debt causes the domestic real interest rate to rise above the prevailing world real interest rate in the short run although the two rates must coincide in the long run.

The model of the almost small open economy differs from the small open economy model in two crucial aspects. First, trade flows are modeled explicitly and relative prices (the real exchange rate) matter in determining whether the economy runs a trade balance deficit or surplus. Second, imports enter the economy as intermediate inputs in the production process for final output and, while final output can be consumed, invested or exported to service foreign loans as before, the world price of final output at any point in time now depends (inversely) on the quantity being exported.

Output (or GDP) per worker can be written as the following monotonic increasing and strictly concave function of capital per worker and imports per worker:

$$q = f(k,m), \text{ where } f_k, f_m > 0; f_{kk}, f_{mm} < 0; \text{ and } f_{kk}f_{mm} - f_{km}^2 > 0$$

Under perfectly competitive conditions all factors are paid their marginal products so the following three relationships define the domestic real interest rate (marginal productivity of capital), the real exchange rate and the real wage:

$$f_k(k_t, m_t) = r_t$$
$$f_m(k_t, m_t) = p^*_m / p_t$$
$$f(k_t, m_t) - k_t f_k(k_t, m_t) - m_t f_m(k_t, m_t) = w_t$$

Notice that, unlike in the simple one sector neoclassical growth model of section 2, the marginal productivity of capital now depends on two variables—the capital–labour ratio and the imports–labour ratio. If capital must earn an after tax rate of return equal to the world real interest rate r^* in the long run this no longer uniquely pins down the capital intensity of production.

The world price of imports is exogenously determined at p^*_m, but the world price of exports depends upon the quantity of exports per worker so that $p_t = p(x_t)$. For simplicity, the export demand schedule has constant elasticity $\eta > 0$, and it shifts outward through time at rate n, the exogenous rate of growth of the world economy. The temporal terms of trade or real exchange rate is then p_t / p^*_m, which is the inverse of the marginal productivity of imports and depends (inversely) on the quantity of per capita exports.

The balance of payments constraint states the usual condition that the sum of the trade account and the service account equals the increase in the country's net external credit position (or reduction in its net foreign indebtedness). Writing the constraint in per worker terms we have:

$$p(x_t)x_t - p^*_m m_t - r^* p_t e_t = -(p_{t+1} e_{t+1} - p_t e_t)$$

where e_t represents the value of the country's net indebtedness per worker in units of domestic output. In a steady state net indebtedness per worker will be constant, as will the world price of exports p_t (and thus the real exchange rate). Then the balance of payments constraint simplifies to:

$$p(x)x - p^*_m m - r^* p(x)e = 0$$

where time subscripts have been omitted because variables are constant in the steady state.

In this model the international capital market works perfectly to equilibriate real after–tax rates of return on alternative forms of savings. We assume that the economy as a whole is a net debtor and in the risk–free, perfect foresight environment posited here there are no gains from international portfolio diversification. Thus, domestics own only two assets, physical capital and government debt, and they choose how much to borrow abroad to finance domestic capital formation. Let τ_k be the *ad valorem* tax on all capital income generated within the economy whether it be financed at home or abroad. Then the after tax rate of return available to domestic savers is $(1-\tau_k)f_k(k_t, m_t) = r_{bt}$. But since capital formation can also be financed externally at the world real interest rate r^* the following relationship ensures that entrepreneurs fully exploit all profitable investment opportunities using the international capital market $[(1-\tau_k)f_k(k_t, m_t)+1]p_t/p_{t-1} = 1 + r^*$. The left hand side is the after tax return on a one period, externally–financed investment; and the right hand side is its cost. Combining these two conditions to eliminate $f_k(\cdot)$ we arrive at the following relationship between the real exchange rate, the real after tax rate of return on domestic savings, and the world real interest rate:

$$(1 + r_{bt})p_t/p_{t-1} = 1 + r^*$$

Thus, the real after tax rate of return on domestic savings will exceed (be below) the world real interest rate whenever the real exchange rate is decreasing (increasing), but in the steady state $r_b = r^*$.

Since the capital income tax drives a constant wedge between the domestic real interest rate and the real after tax rate of return on savings such that $r_t = r_{bt}/(1-\tau_k)$, it is clear that in the steady state the domestic real interest rate will exceed the world real interest rate by the proportion $\tau_k/(1-\tau_k)$.

Suppose that the economy is initially in a steady state equilibrium in which the following seven equations hold

$$(1 - \tau_k)f_k(k,m) = r^*$$

$$p(x)f_m(k,m) = p^*_m$$

$$f(k,m) - kf_k(k,m) - mf_m(k,m) = w$$

$$(r^* - n)e = x - m/p(x)$$

$$(1 + n)a = w - \tau - c(w - \tau, r^*)$$

$$a = k + g - e$$

$$(r^* - n)g = \tau + \tau_k r^* k$$

The last three equations are the assets–savings relation, the capital market equilibrium condition, and the government budget constraint respectively. These equations are identical to those used in the simple, one sector model of the small open economy. The only difference here is that we have introduced a distortive tax on capital income as well as a lump–sum tax on labour income as alternative means of servicing the steady–state government debt when $r^* > n$.

Now consider the long run implications of a reduction in lump–sum taxes on the current young generation which results in a permanent increase in the steady state level of government debt.[7] Differentiating this system totally with respect to a small change in g and solving for the impacts on w, k and x, the following comparative statics effects are obtained:

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[7]Insofar as the short run effects are concerned, it can be shown that an increase in government debt in period t benefits a member of the young generation in period t for two reasons: his or her after tax wage increases and he or she earns a higher return on savings (because the domestic real interest rate rises above the prevailing world real interest rate in the short run as the real exchange rate deteriorates). Since it takes time for the home and foreign real interest rates to coincide (ignoring the wedge created by the tax structure) there is the possibility that some generations of workers employed immediately after the increase in government debt may also benefit from it, but this depends on parameter values. In the small open economy case only the current young generation benefits and all future generations suffer.

$$(8.7) \qquad d(w-\tau)/dg = -(r^* - n)\left\{\frac{\Delta}{\nabla} - \frac{\alpha}{\eta m \nabla}\left[f_m f_{kk} - f_{km}\left[(r^* - n) + \tau_k r^*\right]\right]\right\}$$

$$(8.8) \qquad dk/dg = -(r^* - n)\left\{\frac{\alpha f_{km}}{\eta m \nabla}\left[1 + (r^* - n)(1 - c_w)\right]\right\}$$

$$(8.9) \qquad dx/dg = (r^* - n)\Delta\{1 + (r^* - n)(1 - c_w)\}/\nabla$$

where $\Delta = f_{kk} f_{mm} - f_{km}^2 > 0$, $\alpha = m/px > 0$ *and* $\nabla = \Delta(1 - \alpha/\eta) - \frac{\alpha}{\eta m}\left\{f_m f_{kk} - (r^* - n) f_{km}\left[1 - (1 - c_w)(\tau_k r^* + m\Delta)\right]\right\} > 0$.

We conclude from (8.7) that if the economy is dynamically efficient (i.e. $r^* > n$) an increase in g will reduce the steady state after tax real wage and the steady state lifetime real income of an individual, just as in the small open economy case. However, further manipulation of (8.7) reveals that the expression in braces is greater than one whenever $r^* > n, f_{km} \geq 0$ and η is finite. Therefore the steady state burden of government debt will exceed $r^* - n$ in the almost small open economy case. *M*oreover, from (8.8) if $r^* > n$ and $f_{km} > 0$ an increase in g also reduces the steady state per capita capital stock, so government debt crowds out some domestic capital formation unlike in the small open economy case. Finally, from (8.9) if $r^* > n$ an increase in g increases the steady state level of exports. However, in the almost small open economy an increase in exports necessitates a reduction in their price so the steady state terms of trade (real exchange rate) deteriorates.

In the limit as $\eta \to \infty$ we obtain the small open economy results of Persson (1985). In particular, $d(w-\tau)/dg$ approaches $-(r^* - n)$, dk/dg approaches zero, and dx/dg approaches $(r^* - n)[1 + (r^* - n)(1 - c_w)]$. However, when $r^* > n$ and η is finite there is a hidden cost of government debt; in the almost small open economy government debt causes a permanent

deterioration in the real exchange rate and a permanent reduction in the pre–tax real wage.[8] Moreover, if $f_{km} > 0$ government debt displaces some of the steady–state capital stock and this causes an additional cost whenever there are taxes on capital income.

So far we have assumed that the economy is dynamically efficient so that $r^* > n$. However, if $r^* < n$ then according to the government budget constraint the steady state will feature lump sum *transfers* to the young generation—transfers that are larger the larger is τ_k. An increase in g under these circumstances (caused by an increase in transfers to the current young) will result in a further increase in transfers to the young in the long run so steady state welfare increases. Moreover, if $r^* - n > -1/(1 - c_w)$ it follows from (8.8) that k will increase (if $f_{km} > 0$), and it follows from (8.9) that x will fall. Thus government debt encourages domestic capital formation and improves the steady state terms of trade. These are two additional sources of benefit from government debt that are not present in the small open economy when $r^* < n$.

8.10 Conclusion

In a small open economy in which agents have perfect foresight, are finite lived, and there are no bequests, the steady state burden of government debt is $r^* - n$ whether or not there are taxes on capital income (provided that $\tau_k \geq \tau_k^*$). This follows because government

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[8]The key to understanding this result is to note that for any p_m/p the economy's real value added per worker function can be written as $v(k; p_m/p) = f(k,m) - (p_m/p)m$, where m satisfies the condition $pf_m(k,m) = p_m$. Inverting this latter condition gives $m(k,p_m/p)$. The real value added per worker function is the open economy analog to the standard real output per worker function used in the closed economy (or small open economy) model. It's dual is the factor price frontier which specifies combinations of the real wage and the real interest rate that the economy can support for any given p_m/p. If some event such as an increase in government debt causes p_m/p to fall it shifts the factor price frontier inward so the economy can support a lower real wage for any given real interest rate. Since the real interest rate must equal r^* in the long run an increase in government debt must cause the real wage to fall.

debt increases the country's net foreign indebtedness rather than crowding out the domestic capital stock. In fact, since government debt reduces steady state non–human wealth making future generations poorer, it reduces the pool of private savings and therefore has a magnified effect on net foreign indebtedness.

If $r^* < n$ there is a steady state blessing rather than a burden to government debt. However, instead of rescuing the economy from excessive capital accumulation as in the closed economy, government debt in the small open economy exploits the fact that the economy's growth rate of real output exceeds the rate of return the rest of the world demands to lend to it, and private individuals fail to take advantage of this situation because they have finite lives. Government debt, in effect, circumvents the constraint imposed on private sector behavior by having finite lived agents. Through government debt policy the economy as a whole is able to operate as if it were infinitely lived. But in a world where $r^* < n$ and r^* is exogenous, this situation is fundamentally unstable since the economy's consumption opportunities are boundless.

In a world where countries are linked by a highly efficient international capital market there is limited scope for real interest rate differentials to persist between countries; the existence of country risk premia and cross–country differences in the rate of capital income taxation provide part of the explanation, whereas differences in inflation rates and the existence of non–tradeables do not. Even if the world consists of countries that are almost small (i.e., countries whose exports are perceived to be imperfect substitutes for the tradeables of other countries), real interest rate differentials between countries will be eliminated in an efficient international capital market. However, when countries are almost small, government debt causes a permanent deterioration in terms of trade (real exchange rate) so the steady state burden of government debt exceeds $r^* - n$.

Returning to the case of the small open economy with $r^* > n$, the question arises whether government debt entails a steady state burden even though people willingly hold it at an after–tax rate of return below the economy's growth rate of real output. The answer is in

the affirmative. In the small open economy with finite lived agents and perfect foresight, government debt does not crowd out the domestic capital stock but rather increases net foreign indebtedness so the steady state burden will be $r^* - n$. Since domestic inflation can drive the real after tax rate of return on domestic savings below the world real interest rate r^*, it can also create a situation where $r_b < n < r^*$. Therefore, government debt can involve a steady state burden even though people are willing to hold it at an after–tax rate of return below the economy's growth rate of real output.

9 SUMMARY AND CONCLUSIONS

9.1 Introduction

There are five basic questions that we have addressed in this essay: (1) what determines real rates of interest in the steady state; (2) what can explain the movements in real rates over the last 20 years; (3) can government policies in a small open economy influence rates; (4) do after–tax real rates below the real economic growth rate indicate that government debt should be increased and (5) what are the implications of recent interest rate behavior for government policy. In sections 2 through 8 we have examined aspects of these issues within the context of specific economic environments: In section 2 we developed a growth model that was helpful in identifying issues associated with the steady state rate of interest, the effects of government on this rate, and the possibility of dynamic inefficiency. Sections 3–8 fleshed out specific aspects of that model to provide guidance to modeling possible values of key parameters first in the context of a certain world (sec. 3), next with uncertainty (sec. 4), then with incomplete markets (sec. 5) and finally with other market participants—firms (sec. 6), government (sec. 7) and the rest of the world (sec. 8). The extent that the special assumptions of each section addressed these five questions were noted in the summaries to the various sections. In this section we want to bring together the elements of all the sections insofar as they bear on the questions of this essay.

Before looking at each question individually there is a general remark to be made on the issue of measuring interest rates. There is not just "the" interest rate. There are a variety of interest rates each with its own importance and role to play in the economy. Shocks to the economy or of policy initiatives will alter relative yields, and no single interest rate can serve as a complete guide to the response of the economy to the shock or policy action. There are two rates of interest that we shall most concern ourselves with in this summary: the risk free rate and the cost of capital. The former corresponds to the real return on short term default free debt such as government treasury bills if these were denominated in

terms of commodities rather than money. The latter is the real cost of new funds to a composite of the entire productive sector of the economy. This differs from the return on equity first because it includes not only equity but debt of the productive sector, second because it incorporates implicit costs of marketing new financial securities, including shadow prices of rationing and other market distortions, and third because it takes into account indirect taxes imposed on the productive sector. In other words, the cost of capital is meant to correspond to the marginal product of capital and other productive assets. Other rates of interest that have a bearing on the economy that will play a peripheral role in our discussion in this section are the long term rate of interest on default free debt, the return on equity, the world (risk free) rate of interest and the after tax return on the market portfolio. This latter rate corresponds to the cost of capital less the "cost" wedge to domestic residents associated with taxes on capital, marketing costs of financial intermediation and/or incomplete markets. With these prefatory remarks, we are now ready to address the questions raised in the introduction of this essay.

9.2 The Determination of Interest Rates in the Long Run

A truism in economics is that everything depends upon tastes, technology and the economic environment. Interest rates are no different. Consider tastes first. There are three elements associated with tastes that appear to matter for the determination of interest rates. The first of these is the rate of time preference—the extent an agent values a unit of consumption now relative to a unit of consumption in the future. The greater the rate of time preference the greater the steady state rate of interest. The second element is the length of the planning horizon of an agent. The longer this horizon the longer the benefits accrue from a temporary reduction in consumption now and therefore the greater the incentive to accumulate assets. In the aggregate this greater accumulation of assets reduces steady state interest rates. The third element of tastes that affects interest rates is, technically, the degree of curvature in the (instantaneous) preference function. Preference characterized by a great deal

of curvature are said to exhibit a high level of relative risk aversion, or alternatively, a low level of intertemporal substitution. The level of relative risk aversion need not affect the steady state rate of interest but it will affect the agent's response to variations in future income. In particular, for a given level of income volatility, the greater is the level of relative risk aversion, the lower will be the cost of capital to the firm. The reason for this is that the high risk aversion induces precautionary saving to avoid those periods when income and consumption would otherwise be low. This additional savings implies greater capital accumulation and therefore a lower cost of capital.

Technology can be viewed as a part of the environment households confront in making their decisions on consumption and asset accumulation. The first element of technology that exerts an influence on the steady state rate of interest is the rate of technological change. In our analysis, the greater was the rate of growth in labor productivity, the greater the steady state rate of interest. This is because the innovations tend to decrease the effective capital–labor ratio over time and therefore increase the marginal product of capital relative to what it otherwise would be. The same argument would hold for a greater rate of population growth which could be treated as either a technological or a taste parameter.

The second element of technology that has an influence on real interest rates is through its effects on the distribution of income. In an overlapping generations framework, for instance, any innovation (including changes in the price of intermediate goods) that increases the relative share of output going to the young, who have higher propensities to save than the old, will increase the rate of capital accumulation. Ceteris paribus, this lowers both the marginal product of capital and rate of interest in the steady state. If, on the other hand, different intragenerational tastes were admitted, the point would remain, that the greater the amount of resources going to those with relative low propensities to consume, the lower interest rates will be.

The third element of technology that will influence the average level of interest rates is the distribution of the technological shocks that impact the economy. In the consumption based model of asset pricing used in our analysis, an increase in the variability of income and consumption as measured by the mean preserving spread will decrease the risk free rate of interest but has ambiguous effects on the cost of capital to firms. The first of these effects is because risk averse agents will attempt to avoid the increased uncertainty attached to the risky titles to income. The ambiguous effect on the cost of capital is because the increased desire for precautionary savings may be more than offset by the less attractive option of holding the increasingly uncertain titles to income. A decrease in the expected rate of growth of income has effects similar to those of an increase in income volatility: precautionary savings tends to rise to offset the upcoming bad times but, for risky capital assets, this demand is tempered by the decreased expected rewards from saving in this form. Finally, if the increased volatility or decreased expected income growth was concentrated on labor income, then the cost of capital would decline as capital now provides a *relatively* more attractive source of future income.

As is clear in the previous paragraph, when dealing with an uncertain world, interest rates on different types of securities need not move together. In the case of increased uncertainty associated with capital income, risk free rates could fall and, if households were not especially risk averse, yields on risky assets could rise. This phenomenon of rates moving in opposite directions was also pointed out in a certain world with the term structure: An anticipated increase in wage income in the future can cause short term interest rates rise and long term rates to fall. Because of these types of phenomena, the policy maker must be cautious in setting policy on the basis of one specific interest rate.

We have also considered three elements of what we shall call the economic environment that also influence the level of interest rates. These are the time structure of endowment income to agents, the role of government and the completeness of markets. The effect of the timing of endowment income was discussed in section 3. The more distant in the planning horizon that exogenous income is received the greater will be the rate of interest.

This is because capital accumulation serves as a method of smoothing the flow of consumption in the face of changes in income. If income is received late in life, the less necessary it is to accumulate capital and therefore the greater the rate of interest.

The effect of government on interest rates is pervasive. First, taxing capital income drives a wedge between the cost of funds to firms and the rate of return agents use in their economic calculus. Increases in the tax rate on capital drives up the cost of capital to firms. Depending on whether these taxes are direct or indirect, the observed market rate may rise or fall but it is the total capital tax rate that matters for the cost of capital.

Second, the existence of government debt will generally cause rates to be greater than they otherwise would be, but a variety of qualifications must be attached to this conclusion. First of all, such a conclusion rests on a specific distribution effect, namely that, at the time the debt was issued, the tax liability falls on future generations that are not economically linked to the recipients of the initial transfer. If they were linked it is possible for households to undo the transfer from one group to another. Secondly, the taxes raised to service the debt must not generate increased consumption uncertainty, which would tend to reduce returns on assets as agents increased their precautionary savings in the form of relatively more certain income sources. Third, if there are "liquidity services" provided by government debt, these services cannot be perfect substitutes for those provided by financial intermediaries. If they were perfect substitutes, then financial intermediaries could undo the effects of government debt issue. Finally the issuance of debt should constitute a decrease in the net worth of the government for, if this net worth is unchanged, the additional revenue generated by the capital purchased will pay for the debt servicing and leave private consumption patterns unaffected.

The third way the government can influence interest rates is by financial regulatory policy which affects the wedge between the interest rate facing the household and the cost of capital to the firm. A more competitive intermediation industry will ultimately reduce the cost of capital and increase interest rates received by households. Finally, long run effects of inflation or monetary policy will operate on the spread between liquid and illiquid securities:

increases in anticipated inflation tend to increase this spread.

The final aspect of the economic environment that we emphasized was the incompleteness of markets. This was introduced because, in the consumption based model, the spread between the risk free rate and the cost of capital is too large to be explained by the existence of uncertainty alone. Incomplete markets resolves this anomoly, first by rationalizing monetary services of liquid assets, second by acknowledging the inability to completely diversify and third by recognizing the role of flexibility as a positive attribute of some assets. To the extent that asset and insurance markets are incomplete, market interest rates will be lower, and implicit interest rates on costly to trade assets will be higher, than otherwise. Because of principle–agent and other asymmetric information problems it is likely that many of the funds obtained by firms involve trades in the more costly to operate markets. Thus, the cost of capital will be greater, and the interest rate on liquid risk free securities less than otherwise would be the case if markets were more complete. Further, to the extent that households cannot diversify completely, an income tax/transfer mechanism reduces individual specific risk and as such increases the rate of interest on risk free securities.

9.3 The Recent Behavior of Interest Rates

One application of our analysis is an explanation of the behavior of interest rates over the last two decades. Briefly the real rate of interest on short term marketable securities was historically low during the 1970's and historically high during the 1980's. Movements in the cost of capital are more difficult to assess but do not appear to have exhibited such wide fluctuations. Our analysis suggests three possible explanations for these movements: First, changes in the level of uncertainty, in part associated with the oil shocks in 1973–4 and 1979–80; second, the inflation policies of the U.S. monetary authorities and the subsequent financial deregulation and financial innovation; and third, short run monetary policies. We consider these in turn.

The oil shock of 1973 was unanticipated and the response of governments and individuals to it was one best characterized by panic. Price controls were imposed, rationing of oil was common, and a hightened sense of uncertainty about the level of future income were the order of the day in North America and most of the developed economies. We have argued that an increase in uncertainty will lead to a "flight to quality", that is, an increase in demand for risk free securities, and a decrease in the return on these securities. We have, however, also argued that, on *a priori* grounds, increased uncertainty will have an ambiguous effect on rates on risky securities themselves, and as well, the technological effects of an exogenous increase in the price of intermediate goods provides no clear implication for the cost of capital. Thus in the 70's our model is consistent with the fall in the risk free rate and an increase in the risk premium. The shock in the late 70's probably reinforced this move. However, with the collapse of oil prices and with the increased confidence following the 1982 contraction, we would argue that by late 1982 uncertainty was reduced and expectations of the level of future income rose. These conditions would tend to reverse the effects on the risk free rate. Furthermore, because of the depth of the 1982 contraction, the expected rate of growth of income was probably greater than in previous periods and may account for the historically high real rates in the mid–1980's. If this is the case, then the high current rate can be expected to be transitory.

In addition to shifts in risk premiums over these two decades there were shifts in what we have termed liquidity premiums that reinforced these moves in interest rates. The most important factor influencing these premiums was the behavior of inflation, which rose to historically high levels during the 1970's and the early 1980's. We argued in section 5 that increases in inflation tend to depress measured real rates of interest on money substitutes such as government debt. Thus we would expect rates on these assets to be low until about 1981 and then return to historical levels about 1983. Furthermore, it appears that there have been permanent effects from the high inflation period. In particular, the inflation, in combination with the financial regulation of that period, seems to have generated a burst of financial

innovation that improved the liquidity in a number of markets, thus providing a greater number of substitutes for previously scarce liquid assets. This led to an increase in the real interest rate on previously liquid risk free securities such as governments and easily marketable corporate paper. This increase was further augmented by the decrease in regulatory restrictions on major financial institutions, further increasing the relative supplies of liquid securities. This could explain why real rates on liquid risk free securities have remained at historically high levels since 1983 and can be expected to persist at these levels. On the positive side, however, it does suggest that the cost of capital has not risen in line with the increase in the risk free rate of interest and may, in fact, have decreased. This is because these arguments suggest that the premium has decreased as a result of the financial innovations that have occurred.

Changes in the spread between the risk free rate of interest and the cost of capital can explain much of the fall in risk free rates prior to 1980 and after 1982. We have no good long term explanation of the intervening period, from late 1980 through mid–1982, when inflation was still high and investment demand weak. Our tentative explanation for this period must therefore rely on short run monetary policy. Monetary policy in this interval not only can be viewed as contractionary in the traditional sense but also U.S. monetary policy was widely perceived to be in a transition from one type of policy regime to another that appeared less willing to tolerate inflation. During such transitions rational expectation forecasts are especially difficult. As a result, unanticipated monetary contractions likely prevailed, suggesting higher real interest rates than otherwise would be the case.

9.4 The Open Economy

Capital markets throughout the world are becoming increasingly integrated. As such domestic factors will play a correspondingly smaller part in influencing the level of domestic interest rates. Nonetheless, we find that, even under the assumptions of a small open economy and perfect capital markets, government tax policies can still influence the level of interest

rates domestically. This is because it is after–tax real rates of interest that are equalized rather than interest rates themselves and, by international convention, governments have the ability to tax all income generated within its borders. Thus, increases in the domestic tax rate above the world rate of capital taxation will increase the cost of capital in the host country. Decreases in the capital tax rate below the world rate, however, will not decrease the cost of capital because virtually all nations provide their nationals the use of foreign tax credits. Thus, lowering the capital tax rate below the world rate simply transfers revenue from the host country to treasuries in the rest of the world.

A second mechanism by which governments can alter the real after–tax rate of interest is through inflation policy. This is because the tax base for foreign governments includes capital gains and losses from currency changes whereas these are not included in the host government's tax base. As a consequence, increases in domestic inflation lead to a decrease in the after–tax real rate of return residents receive from saving while decreases in domestic inflation will increase their return.

World and domestic real rates can also diverge in the short run if government debt causes a change in the real exchange rate. As an example we analyze the "almost" small open economy where the home country has market power in its exportable good. An increase in government debt in this situation will cause a temporary increase in the domestic real interest rate above the world rate, a permanent deterioration in the real exchange rate, and a permanent reduction in the pre tax real wage. Government debt will also crowd out some of the domestic capital stock and this carries an additional burden when there are taxes on capital income. The burden of the debt is therefore greater than it would be in the small open economy case.

9.5 Government Borrowing

If the cost of capital is below the rate of output growth in a certain world, then a reduction in the capital stock is desirable. This is because the return to society from the last unit of capital per worker is less than the cost of maintaining that capital in the future: capital

acts as a drain on resources available for consumption and desirable government spending. In such a situation the issuance of government debt that crowds out capital can be viewed as desirable since it removes a net cost to society. In an uncertain world the situation is similar: If capital in all states of nature generates less goods than are used to maintain the capital stock too much capital is in existence and crowding out some of the excess capital is welfare improving.

While such a possibility is interesting to consider, the empirical data is overwhelming in suggesting that economies of record have never been in a situation where there has been "too much" capital: Capital on an annual basis consistently has contributed more output for consumption and government spending than the resources it has absorbed to maintain and/or increase itself on a per capita basis, i.e. the value added by capital less gross investment on an annual basis has consistently been positive in the developed countries, at least over the last 60 years. The question of whether it is desirable as a practical matter for the government to issue debt simply to crowd out existing capital is thus resoundingly answered in the negative: Economies are not dynamically inefficient nor would we expect them to be any time in the foreseeable future.

The above observation has an obvious bearing on structuring government financing when the real rate of interest on government debt is less than the rate of output growth, namely that the social cost of government debt is greater than the interest rate on those funds. To see this, note simply that if government debt crowds out private capital, then it is the expected return on that capital that is lost to society and this expected return is greater than the "risk free" rate of interest that the government pays on its debt.

There is one qualification to this result. If the economy is small and the cost of funds to government is less than the rate of output growth, then issuing debt may not crowd out any capital because it does not affect the world rate of interest. What occurs in this case is that non–residents purchase the debt, leaving the capital stock unchanged, or if residents purchase

it, non–residents purchase the capital that residents sold to obtain the government debt, once more leaving the capital stock unchanged.

As a practical matter, one should not put too much stock in the government's ability to exploit the rest of the world in this manner. First of all, there is the possibility that the economy may grow at a slower rate in the future, in which case, to maintain or decrease the stock of debt per capita would require future generations to give up real resources to service the debt. Secondly, one can never be sure that, at some time in the future, the cost of funds to the government will not rise. Indeed, if the government of one small economy perceives an incentive to issue debt, then other small economies could be expected to also perceive this same opportunity, leading all governments to issue debt which would crowd out capital. Such strategic behavior would leave future generations in all countries worse off.

This is not to say that governments should never borrow. The point is instead that they should be aware of the true opportunity cost of their actions. These costs are more than simply the cost to the government of borrowing funds.

9.6 Governments and Interest Rates

Government actions throughout the world certainly had an effect on interest rates over the last two decades. Decreases in capital taxation rates may have contributed to the investment boom of the 1980's. The low real rates on government debt and other relatively riskless securities can be attributed to the inflationary policies of many of the major central banks in the 1970's. Even the historically high rates of this decade may trace their origin to that inflation, which provided a powerful incentive for innovation by financial institutions. These innovations increased the relative supply of liquid securities, decreasing the interest rate spread between liquid and relatively illiquid securities and, in the process, improved the operations of markets. The move toward deregulation on the part of a number of governments

further enhanced the efficiency of financial markets and brought down the cost of capital to a variety of potential market participants. Under this interpretation the increase in the cost of funds to government and rates of interest on risk free securities is the manifestation of desirable developments in financial markets and should not necessarily be resisted. While it is true that, with a limited supply of funds, changes in interest rate spreads will imply some demanders will be worse off, it strikes us that movements toward improved markets is better than the alternative of resisting change. In any case, once this process has begun it would require draconian measures to return to the "good old days" of increased economic uncertainty, arbitrary interest ceilings, and credit rationing.

On a less positive note, some of the increase in overall interest rates may be due to the increase in government debt that has crowded out some real capital formation. If this policy was undertaken specifically to redistribute resources to groups with higher propensities to spend, then it would be reasonable to assume that the capital investment and interest rate consequences were included in the social calculus. Our own hunch is that the growth of debt was not so much a reasoned decision regarding intergenerational redistribution so much as a opportunistic response to the low interest rates of the 1970's. In a small economy there is an opportunity to exploit the rest of the world at little cost to domestic residents when governments can borrow at rates below the growth rate of output. But since all countries face a similar incentive, the aggregate response is such that rates will rise until these opportunities no longer exist. It is not beyond the realm of possibility that such actions were at the heart of the increase in government debt from the middle of the 1970's.

9.7 The Future

We do not know what real rates of interest will be in the future. What we have detailed in this essay simply indicates some of the forces at work that have influenced interest rates recently and may continue to affect them in the future. It nonetheless may be useful to

conclude this essay by briefly noting how we might link the future behavior of interest rates to the forces that have existed in the past.

The forces that have had an important influence on interest behavior over the past two decades have been changes in inflation, changes in the perceived levels of uncertainty, and changes in the efficiency of markets. Central bankers do not appear to be willing to return to the inflationary policies of the 1970's so we do not think that this factor will be a source of change in the near future. We also can hardly anticipate any shifts in the level of uncertainty agents feel about the future. That leaves the increased efficiency of financial markets as a permanent legacy from the last two decades. This has decreased the cost of capital to borrowers who would otherwise have been non–priced rationed in credit markets. It has also increased the cost of funds to other borrowers, most notably governments. We do not see financial markets in the future collapsing, financial innovations being undone, nor the reinstitution of the old forms of financial regulation. It therefore strikes us that governments will continue to face what to them appear to be high historical real rates of interest for the foreseeable future. We hope they take heart knowing that their citizens will, on average, be obtaining funds at rates lower than otherwise would be the case.

There are, finally, two factors that may emerge that will affect future interest rates that did not, as far as we can tell, play a significant role over the period 1970 to the present. The first is the effect of demographic shifts: It may be that life cycle behavior will imply a large increase in savings rates in the near future causing low rates of interest for the next three decades. We believe this is an area requiring further study. The second factor is the continuing growth of government debt. To this point we do not believe that it has had a significant effect on interest rates but recognize that the effects of deficits are cumulative and that the long run effects may be opposite those of the short run. For personal reasons, if not professional ones, we would not wish to advocate continuing growth of the debt only to find out that such a policy does indeed have substantial long run effects on the capital stock, welfare and the level of real interest rates.

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